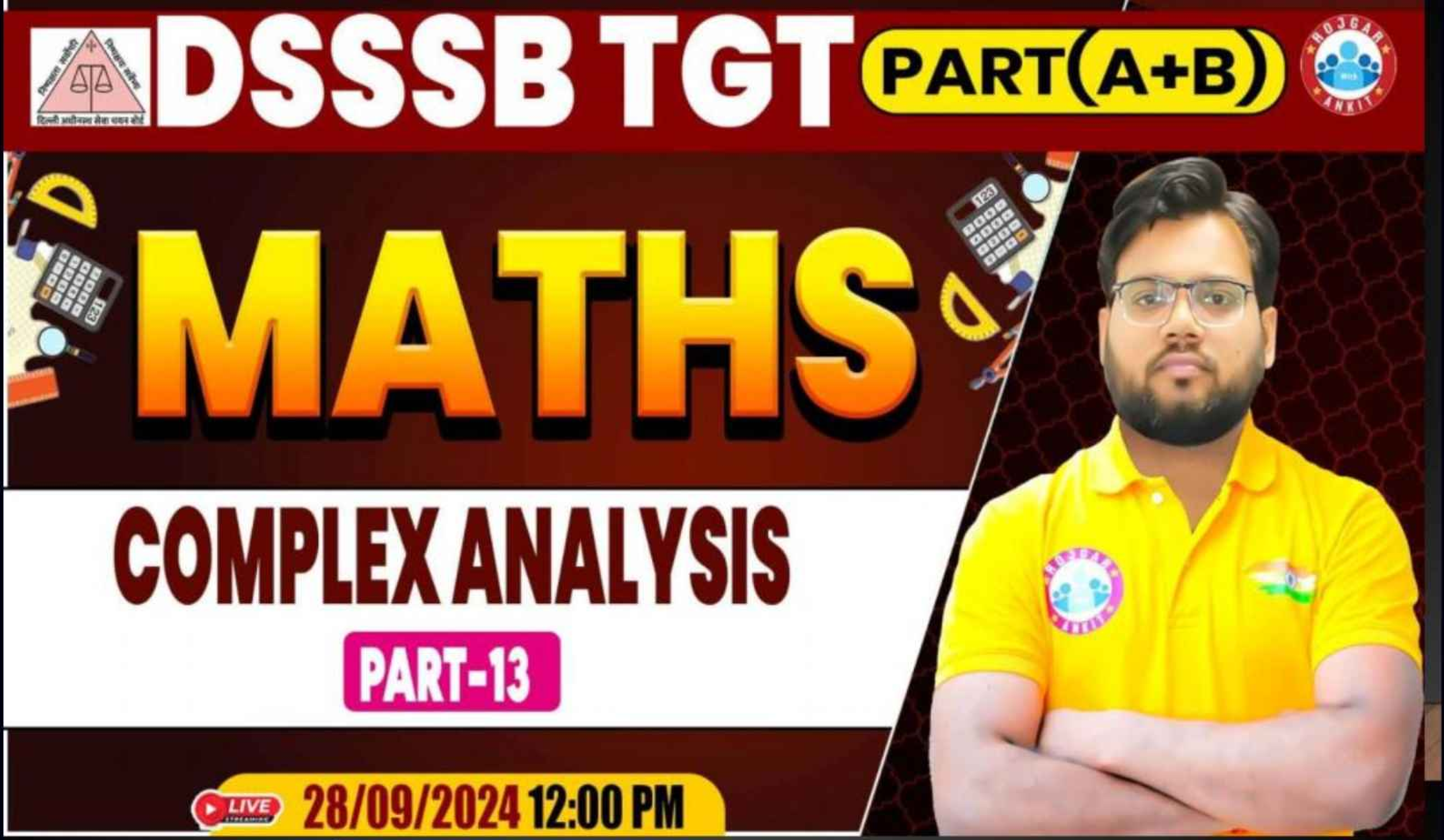
DSSSB TGT
PART(A+B)
ROJGAR
ANKIT
MATHS
COMPLEX ANALYSIS
PART-13
LIVE
28/09/2024 12:00 PM

Complex Sequence: - A function $f: \mathbb{N} \to \mathbb{C}$ is said to be a Complex sequence

$\langle z_n \rangle, \langle \frac{i}{n} \rangle, \langle i^n \rangle$

$\langle i \rangle, \langle i+n \rangle$

Real Analysis $\rightarrow$

$N \longrightarrow R$

$\langle u_n \rangle = \langle u_1, u_2, u_3 \cdots \rangle$

Convergent sequence: - Let $\langle z_n \rangle$ be a sequence which convergent to Z_0 if for given $\varepsilon > 0, \exists m \in \mathbb{N}$ s.t. $|z_n - z_0| < \varepsilon \forall n \geqslant m$.

$\langle z_n \rangle$

$$\langle z_n \rangle \xrightarrow{cgt} z_0$$

$$|z_n - z_0| < \varepsilon \quad \forall n \geq m$$

Algebra ⇒ plus + minus − multi × Div ÷

Note:- (1) sequence $\langle z_n \rangle = \langle x_n + iy_n \rangle$

(2) Algebra of complex sequence is same as that of real sequence. ✓

(3) Sequence $\langle z_n \rangle$ converges to $\underline{z_0}$ and sequence $\langle z_n' \rangle$ converges to z_0'. Then,

(i) $\lambda z_n \pm z_n'$ converges to $\lambda z_0 \pm z_0'$

(ii) $z_n \cdot z_n'$ converges to $z_0 \cdot z_0'$

(iii) $\frac{z_n}{z_0'}$ converges to $\frac{z_0}{z_0'}$

(iv) If $\langle z_n \rangle$ converges to z_0. Then, $|z_n|$

(vi) If $\langle z_n \rangle \xrightarrow{cgt} z_0 \Rightarrow \bar{z}_n \xrightarrow{cgt} \bar{z}_0$

(vii) $\Sigma z_n = \sum(x_n + iy_n) = \sum x_n + i\sum y_n$ then $\sum z_n$ is Convergent $\Leftrightarrow \sum x_n < \infty$ Convergent and $\sum x_n < \infty$ Convergent

$n \in \mathbb{N}$

$\langle i^n \rangle = \langle i, i^2, i^3, i^4, i^5, i^6, \ldots \rangle$

$\langle i, -1, -i, 1, i, -1, -i, 1 \ldots \rangle$

Q. Sequence $\langle i^n \rangle$ is ?

a) Convergent

b) Divergent

c) Oscillating

d) None of These

$\langle i^{4n} \rangle = 1$

$\langle i^{4n+1} \rangle = i$

$\langle i^{4n+2} \rangle = i^2 = -1$

$\langle i^{4n+3} \rangle = i^3 = -i$

Limit neither unique nor ∞. So the Sequence is Oscillating

$\lim_{n \to \infty} \langle u_n \rangle \to$ unique, Exists, finite — cgt

$\lim_{n \to \infty} \langle u_n \rangle \to +\infty, -\infty$ — dgt

$\lim_{n \to \infty} \langle u_n \rangle$ — not Unique $\to$ oscillating

$$\left\langle \frac{i}{n} \right\rangle = \left\langle \frac{i}{1}, \frac{i}{2}, \frac{i}{3}, \frac{i}{4}, \frac{i}{5} \cdots\cdots \right\rangle$$

$$\lim_{n\to\infty} \left\langle \frac{i}{n} \right\rangle \Rightarrow \frac{i}{\infty} = 0$$

$\rightarrow$ finite

$\rightarrow$ Unique

Q. Sequence $\left\langle \frac{i}{n} \right\rangle$ is ?

a) Convergent

b) Divergent

c) Oscillating

d) None of These

Q. Sequence $\langle 1 + i^n \rangle$ is?

a) Convergent

b) Divergent

c) Oscillating

d) None of These

$\langle 1+i^n \rangle \Rightarrow \langle 1+i, 1+i^2, 1+i^3, 1+i^4 \cdots\cdots \rangle$

$\langle 1+i, 0, 1-i, 2 \cdots\cdots \rangle$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$

Not unique

$\langle z_n \rangle$

$\sum z_n$

Ratio Test: $\to \sum z_n$ is a series

Then, $\lim_{n\to\infty} \left|\frac{z_{n+1}}{z_n}\right| = l$

(1) $l < 1 \Rightarrow$ Convergent

(2) $l > 1 \Rightarrow$ Divergent

(3) $l = 1 \Rightarrow$ Test fails

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$

$l < 1 = $ Cgt

$l > 1 = $ dgt

$l = 1 \Rightarrow$ Test fails

$\lim_{n\to\infty} |z_n|^{\frac{1}{n}} = l$

Root Test: $\to \lim_{n\to\infty} \sqrt[n]{|z_n|} = l$

(i) $l < 1 \Rightarrow$ Convergent

(ii) $l > 1 \Rightarrow$ Divergent

(iii) $l = 1 \Rightarrow$ Test fails

Power Series: - A series of the form $\boxed{\sum a_n (z-a)^n}$ is said to be a power series with center at $z = a$.

Radius

(1) If $R = 0$ Then Convergent only for $z = 0$

(2) If $R = \infty$ $\hookrightarrow$ Convergent in $\mathbb{C}$.

z^n है तो $\frac{1}{R}$

z^{2n} है तो $\frac{1}{R^2}$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{z_{n+1}}{z_n} = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$$

Radius of Convergence

Q. $\sum \frac{z^n}{n^2}$; **find radius of convergence and Region?**

~~a) 1, $|z| = 1$~~

~~b) 3, $|z| < 1$~~

$\boxed{\text{c) } 1, |z| \leq 1}$

~~d) 2, $|z| \leq 2$~~

$\sum a_n (z-a)^n$

$a = 0$

$\sum \frac{1}{n^2} \cdot z^n$

$a_n = \frac{1}{n^2}$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{1}{n^2} \right|^{\frac{1}{n}}$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{1}{n^{\frac{2}{n}}} \right| = \left| \frac{1}{n^0} \right| \to 1$

$\frac{1}{R} = 1$

$\boxed{R = 1}$

$|z| < R, \quad |z| \leq 1$

$z = a + re^{i\theta} \to$ Eqn of Circle in parametric form

$z = 0 + 1e^{i\theta}$

$\boxed{z = e^{i\theta}} = \cos\theta + i\sin\theta$

$|z| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

$\boxed{|z| = 1}$

Q. $\sum \frac{n}{6^n} z^n$; find radius of convergence and Region?

a) $2, |z| = 4$

b) $6, |z| < 6$

c) $6, |z| \le 6$

d) $6, |z| \le 3$

$\sum \frac{n}{6^n} \cdot Z^n$ $\quad a=0$

$a_n = \frac{n}{6^n}$

$\frac{1}{R} = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$

$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{n}{6^n}\right|^{\frac{1}{n}}$

$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{n^{\frac{1}{n}}}{6}\right| \Rightarrow \frac{1}{R} = \lim_{n\to\infty} \left|\frac{n^0}{6}\right| \Rightarrow \frac{1}{6}$

$\frac{1}{R} = \frac{1}{6}$

$\boxed{R=6}$

$|z| < R$

$|z| < 6$

$z = a + re^{i\theta}$

$= 0 + 6 \cdot e^{i\theta}$

$z = 6e^{i\theta}$

$|z| = |6e^{i\theta}|$

$|z| = 6\sqrt{1} \Rightarrow 6$

$|e^{i\theta}| = |\cos\theta + i\sin\theta|$

$\Rightarrow \sqrt{\cos^2\theta + \sin^2\theta}$

$\Rightarrow 1$

Q. $\sum \frac{z^n}{n!}$; **find radius of convergence and convergent or Divergent?**

a) n, divergent

b) ∞, convergent

c) $6, |z| \leq 6$

d) n, convergent

$\sum \frac{1}{n!} \times z^n$ $\quad a=0$

$a_n = \frac{1}{n!}$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{1}{(n+1)!} \times \frac{n!}{1} \right|$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{\cancel{n!}}{(n+1) \times \cancel{n!}} \right|$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{1}{n+1} \right|$

$\frac{1}{R} = \frac{1}{\infty+1} = \frac{1}{\infty} = 0$

$\frac{1}{R} = 0 \Rightarrow R = \frac{1}{0} = \infty$

∞, Conv

Q. $\sum n!\,z^n$; Find Radius of Convergence?

a) 0 , divergent

b) ∞, convergent

c) 0 , convergent only for $z = 0$

d) 1 ,convergent only for $z = 1$ ×

$$\sum n!\,z^n \qquad a=0$$

$$a_n = n!$$

$$\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$\frac{1}{R} = \lim_{n\to\infty}\left|\frac{(n+1)!}{n!}\right|$$

$$\frac{1}{R} \Rightarrow \lim_{n\to\infty}\left|\frac{(n+1)\cdot\cancel{n!}}{\cancel{n!}}\right|$$

$$\frac{1}{R} = \lim_{n\to\infty}|n+1|$$

$$\frac{1}{R} = \infty$$

$$\boxed{\frac{1}{\infty} = R}$$

$$R = 0$$

Q. $\sum 2^n \cdot z^n$, **find convergence Region?**

a) $|z| = 2$

b) $|z| < 1$

c) $|z| \leq 1/3$

d) $\boxed{|z| < 1/2}$

$\sum 2^n \cdot z^n$ $\quad a=0$

$a_n = 2^n$

$$\frac{1}{R} = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$$

$$\frac{1}{R} = \lim_{n\to\infty} |2^{\cancel{n}}|^{\frac{1}{\cancel{n}}}$$

$$\frac{1}{R} = \lim_{n\to\infty} 2$$

$$\frac{1}{R} = 2 \qquad \boxed{R = \frac{1}{2}}$$

$|z| < R$

$|z| < \frac{1}{2}$

$z = a + re^{i\theta}$

$z = 0 + \frac{1}{2} e^{i\theta}$

$|z| = \frac{1}{2} |\cos\theta + i\sin\theta|$

$|z| = \frac{1}{2} \times 1$

$$\boxed{|z| = \frac{1}{2}}$$

$\sum n^2 \cdot z^n$ $a=0$

$a_n = n^2$

$\frac{1}{R} = \lim_{n\to\infty} |n^2|^{\frac{1}{n}}$

$\frac{1}{R} = \lim_{n\to\infty} |n^{\frac{2}{n}}| = 1$

$\boxed{R=1}$

Q. $\sum n^2 \cdot z^n$**. Find radius of convergence and Region of Convergence?**

a) $1, |z| < 1$

~~b)~~ $3, |z| < 1$

$\boxed{\text{c) } 1, |z| \leq 1}$

~~d)~~ $|z| \leq 1/4$

$z = 0 + 1 \cdot e^{i\theta}$

$\boxed{|z| = 1}$

$|z| \leq 1$