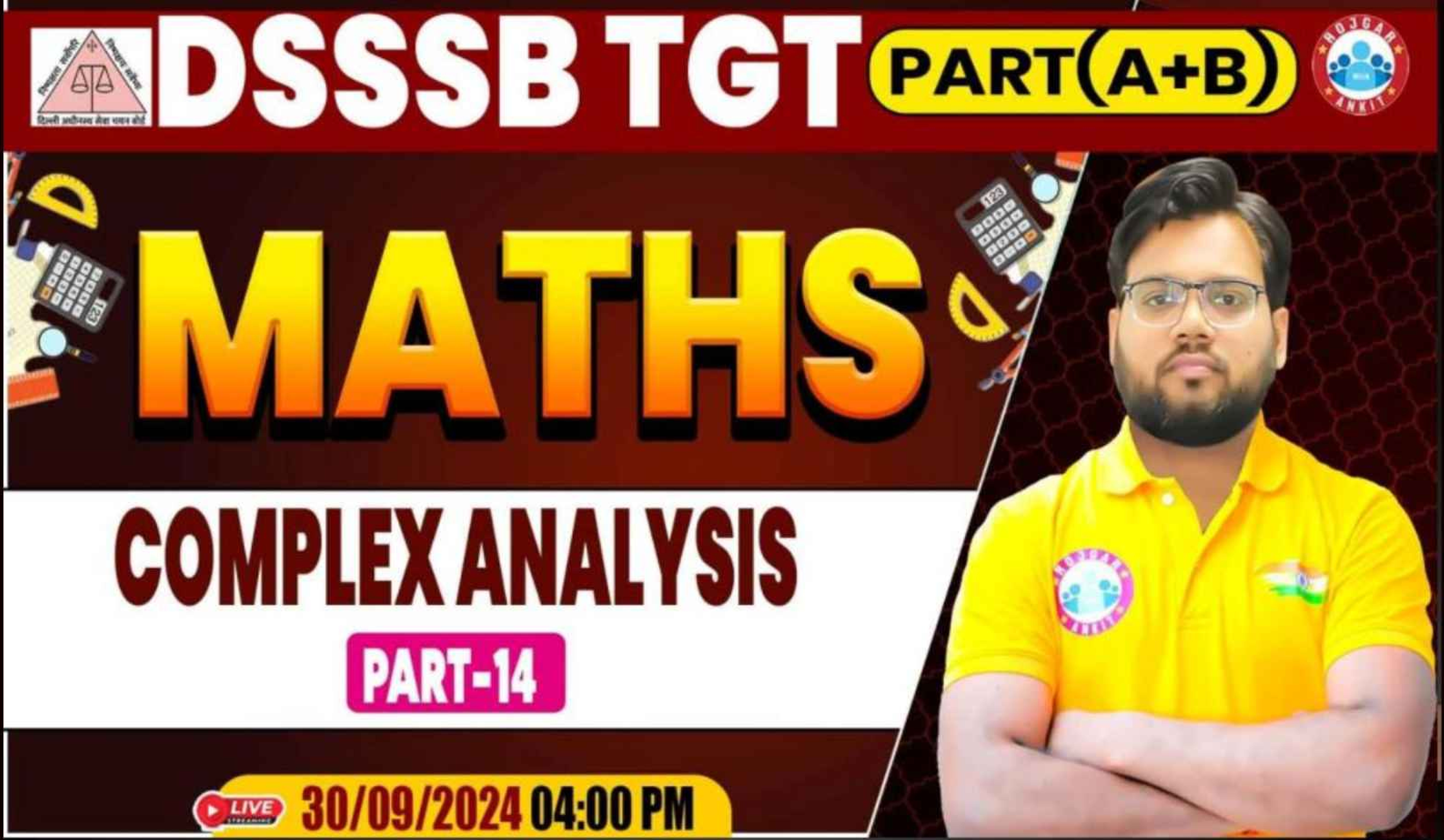

DSSSB TGT
PART(A+B)
ROJGAR ANKIT
MATHS
COMPLEX ANALYSIS
PART-14
LIVE
30/09/2024 04:00 PM

# Complex Function

**Complex Function: - A function** $f: D \subseteq \mathbb{C} \to \mathbb{C}$ **is said to be a Complex Function**

$e^z$, $\sin z$, $\cos z$, $z^2$, $z^3$ - - -

$$z = x + iy$$
$$= a + ib$$
$$= \alpha + i\beta$$

$$z^2 = (x+iy)^2$$

$$\Rightarrow \underbrace{x^2 - y^2}_{\text{Real}} + i\,\underbrace{2xy}_{\text{Imag}}$$

$$e^z = e^{x+iy} = e^x \cdot e^{iy}$$

$$= e^x(\cos y + i \sin y)$$

$$\Rightarrow \underbrace{e^x \cos y}_{\text{Real}} + i\,\underbrace{e^x \sin y}_{\text{Imaginary}}$$

$$\sin z = \sin(x+iy) \Rightarrow \sin x \cdot \cos iy + \cos x \cdot \sin iy$$

$$\Rightarrow \sin x \cdot \cosh y + \cos x(i \sinh y)$$

$$\Rightarrow \underbrace{\sin x \cdot \cosh y}_{\text{Real}} + i\,\underbrace{\cos x \cdot \sinh y}_{\text{Ima}}$$

$$\cos iy = \cosh y$$

$$\sin iy = i \sinh y$$

$$\tan iy = i \tanh y$$

# Complex Analysis

## Trigonometric Function

$$\sin z = \sin(x+iy)$$

$$\cos z = \cos(x+iy)$$

$$\Rightarrow \cos x \cdot \cos iy - \sin x \cdot \sin iy$$

$$\Rightarrow \cos x \cdot \cosh y - i \sin x \cdot \sinh y$$

# Trignometric function :-

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \qquad \operatorname{cosec} z = \frac{2i}{e^{iz} - e^{-iz}}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}, \qquad \sec z = \frac{2}{e^{iz} + e^{-iz}}$$

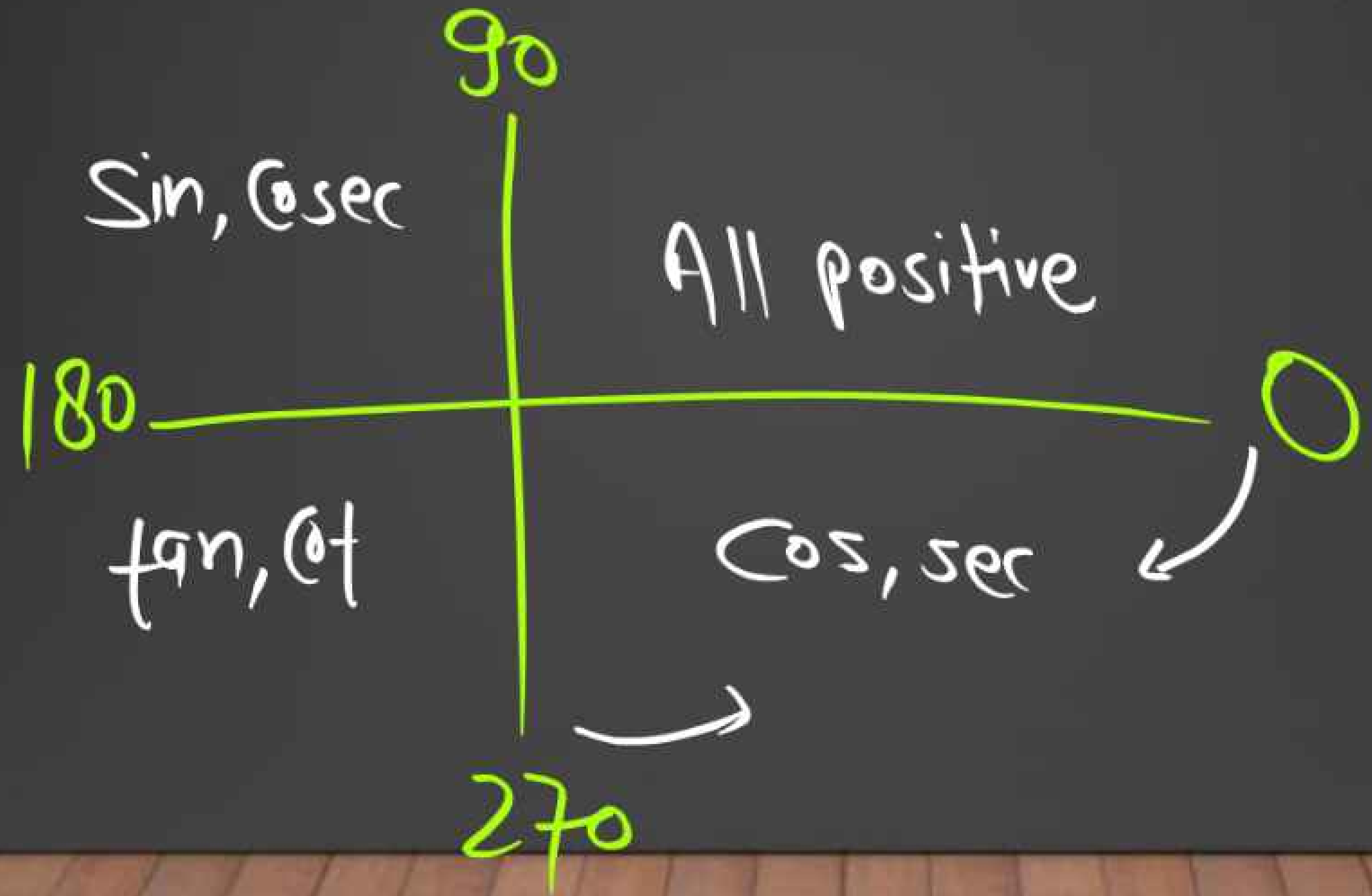
$$\tan z = \frac{\sin z}{\cos z} = \frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}, \qquad \cot z = \frac{i(e^{iz} + e^{-iz})}{e^{iz} - e^{-iz}}$$

## Some Results: -

$$\sin^2 z + \cos^2 z = 1; \quad 1 + \tan^2 z = \sec^2 z$$

$$1 + \cot^2 z = \operatorname{cosec}^2 z; \quad \sin(-z) = -\sin z$$

$$\cos(-z) = \cos z; \quad \tan(-z) = -\tan z$$




$\cos(0-\theta)$

$\cos\theta$

# Hyperbolic Function: -

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\sinh z = \frac{e^{z}-e^{-z}}{2}; \cosh z = \frac{e^{z}+e^{-z}}{2}$$

$$\operatorname{sech} z = \frac{1}{\cosh z} = \frac{2}{e^{z}+e^{-z}}; \operatorname{cosech} z = \frac{1}{\sinh z} = \frac{2}{e^{z}-e^{-z}}$$

$$\tanh z = \frac{\sinh z}{\cosh z} = \frac{e^{z}-e^{-z}}{e^{-}+e^{-z}}; \coth z = \frac{\cosh z}{\sinh z} = \frac{e^{z}+e^{-z}}{e^{z}-e^{-z}}$$

$$\sinh(z_1 \pm z_2) = \sinh z_1 \cdot \cosh z_2 \pm \cosh z_1 \cdot \sinh z_2$$

$$\sin(iz) = i\sinh z; \boxed{\cos(iz) = \cosh z}$$

$$\tan(iz) = i\tanh z; \sinh(iz) = i\sin z$$

$$\cosh(iz) = \cos z; \tanh(iz) = i\tan z$$

$$\cosh^2 z - \sinh^2 z = 1$$

$$1 - \tanh^2 z = \operatorname{sech}^2 z$$

$$\coth^2 z - 1 = \operatorname{cosech}^2 z$$

**Q. The value of** $\cosh^2 z - \sinh^2 z$

$\cosh^2 z - \sinh^2 z$ **का मान-**

**(a)** $\cosh(2z)$

**(b) 1**

**(c)** $\sinh(2z)$

**(d) 0**

$\cosh^2 z - \sinh^2 z = 1$

**Q. The Real part of** $\sin(x+iy)$ –

$\sin(x+iy)$ का वास्तविक भाग है

(a) $\sin x$

(b) $\cos y$

(c) $\sin x \cos hy$

(d) $\sin x \sin hy$

$\sin(x+iy) \Rightarrow \sin x \cdot \cos iy + \cos x \cdot \sin iy$

$\Rightarrow \underbrace{\sin x \cdot \cosh y}_{\text{Real part}} + i \cos x \sinh y$

**Q. What will be the imaginary/part of** $\cosh(\alpha + i\beta)$ **?** $z$

$z = \alpha + i\beta$

$\cosh(\alpha + i\beta)$ **का काल्पनिक भाग होगा?**

**(a)** $\cosh\alpha\cos\beta$

**(b)** $\boxed{\sinh\alpha\sin\beta}$

**(c)** $-\sin h\alpha\sin\beta$

**(d)** $-\cos h\alpha\cos\beta$

$\cos iz = \cosh z$

$\sin iz = i\sinh z$

$\tan iz = i\tanh z$

$\cosh z = \cos iz$

$= \cos i(\alpha + i\beta)$

$= \cos(i\alpha - \beta)$

$= \cos i\alpha \cdot \cos\beta + \sin i\alpha \cdot \sin\beta$

$\Rightarrow \cosh\alpha \cdot \cos\beta + i\underbrace{\sinh\alpha \cdot \sin\beta}_{\text{Imaginary}}$

# Logarithmic function →

$\log(1) = 0$

$\ln(1) = 0$

$$e^z = \omega$$

$$\log e^z = \log \omega$$

General form

$$z = \log|w| + i(\arg w + 2n\pi) \; ; \; n \in Z'$$

Or $$z = \ln|\omega| + i(\arg(\omega) + 2n\pi) \; ; \; n \in Z'$$

Or $$z = \ln|\omega| + i \arg(\omega)$$

# Logarithmic Function:

**Q.** $\log(1+i)$ **is equal to :/log** $(1+i)$ **बराबर है-**

**(a)** $\frac{1}{2}\log 2$

**(b)** $\frac{\pi}{4}+\frac{1}{2}\log 2$

**(c)** $\frac{1}{2}\log 2 + i\frac{\pi}{4}$

**(d) None of these**

$\log m^n$

$n\log m$

$\log(1+i) = \log|1+i| + i(\arg(1+i) + 2n\pi)$

$\Rightarrow \log\sqrt{1^2+1^2} + i\left(\tan^{-1}\left|\frac{1}{1}\right| + 2n\pi\right)$

$= \log 2^{\frac{1}{2}} + i\left(\frac{\pi}{4} + 2n\pi\right)$

$\Rightarrow \frac{1}{2}\log 2 + i\frac{\pi}{4}$

**Q. Find out the all values of $Z$ from this equatio**

$e^z = 1 + \sqrt{3}i$.

a. $\ln(2) - (\pi \div 2 + 2\pi n)i; n \in Z$

b. $\ln(2) - (\pi \div 3 + \pi n)i; n \in Z$

c. $\ln(2) + (\pi \div 2 + \pi n)i; n \in Z$

d. $\boxed{\ln(2) + (\pi \div 3 + 2\pi n)i; n \in Z}$

$e^z = \omega$

$e^z = \log|1+\sqrt{3}| + i\left(\arg\left(\frac{\sqrt{3}}{1}\right) + 2n\pi\right)$

$\Rightarrow \log\sqrt{1+3} + i\left(\tan^{-1}\left|\frac{\pi}{3}\right| + 2n\pi\right)$

$\Rightarrow \log 2 + i\left(\frac{\pi}{3} + 2n\pi\right), n \in Z$

$\Rightarrow \ln(2) + i(\pi \div 3 + 2n\pi), n \in Z$

# Inverse Trigonometric Function:

$$\sin^{-1} z = \frac{1}{i}\ln\left(iz + \sqrt{1 - z^2}\right) = \frac{1}{i}\log\left(iz + \sqrt{1-z^2}\right)$$

$$\operatorname{cosec}^{-1} z = \frac{1}{i}\ln\left(\frac{i+\sqrt{z^2-1}}{z}\right)$$

$$\cos^{-1} z = \frac{1}{i}\ln\left(z + \sqrt{z^2 - 1}\right) \Rightarrow \frac{1}{i}\log\left(z+\sqrt{z^2-1}\right)$$

$$\sec^{-1} z = \frac{1}{i}\ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$

$$\tan^{-1} z = \frac{1}{2}\ln\left(\frac{1+iz}{1-iz}\right)$$

$$\cot^{-1} z = \frac{1}{2i}\ln\left(\frac{z+i}{z-i}\right)$$