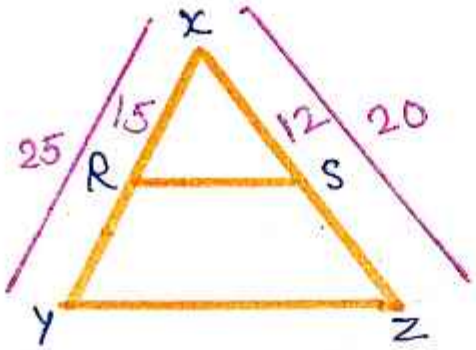
GEOMETRY

THALES THEOREM

Q) R and S are points on sides XY and XZ of ΔXYZ respectively. Also, XR = 15 cm, XY = 25 cm, XS = 12 cm and XZ = 20 cm. Then RS is equal to.

R और S क्रमश: ΔXYZ की भुजाओं XY और XZ पर स्थित बिंदु है। साथ ही, XR = 15 सेमी, XY = 25 सेमी, XS = 12 सेमी और XZ = 20 सेमी। तो RS इसके बराबर है।



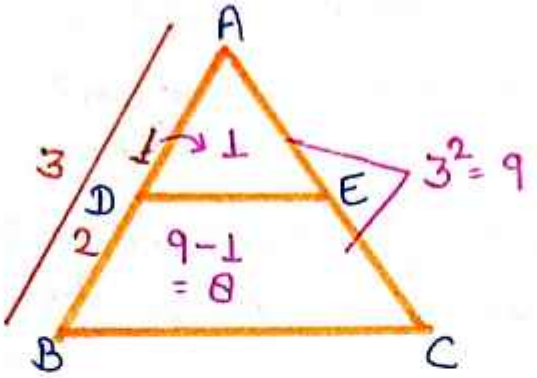
$$\frac{XR}{XY} = \frac{\cancel{15}}{\cancel{25}}\frac{3}{5} \qquad \frac{XS}{XZ} = \frac{\cancel{12}}{\cancel{20}}\frac{3}{5}$$

$$\frac{RS}{YZ} = \frac{3}{5}$$

$$RS = \frac{3}{5}YZ$$

Q) D and E are any points on sides AB and AC of ΔABC. DE is parallel to BC. If AD : DB = 1 : 2 and area of ΔABC is 45 cm^2, then what is the area of quadrilateral BDEC?

ΔABC के भुजा AB और AC पर D और E कोई बिंदु हैं। DE, BC के समानांतर है। यदि AD : DB = 1 : 2 है और ΔABC का क्षेत्रफल 45 वर्ग सेमी है, तो चतुर्भुज BDEC का क्षेत्रफल क्या है?



$9 \rightarrow 45\,cm^2$

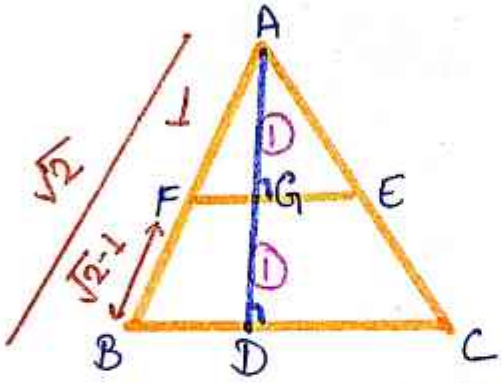
$1 \rightarrow 5\,cm^2$

$8 \rightarrow 8 \times 5$

$40\,cm^2$

Q) In ΔABC, F and E are points on sides AB and AC respectively such that $FE \parallel BC$ and FE divides the triangle into two parts of equal area. If $AD \perp BC$ and AD intersects FE at G, find GD : AG.

ΔABC में, भुजाओं AB और AC पर क्रमशः बिंदु F और E इस प्रकार हैं कि $FE \parallel BC$ है और FE त्रिभुज को समान क्षेत्रफल वाले दो भागों में विभाजित करती है। यदि $AD \perp BC$ है और AD FE को G पर काटती है, GD : AG ज्ञात कीजिये।



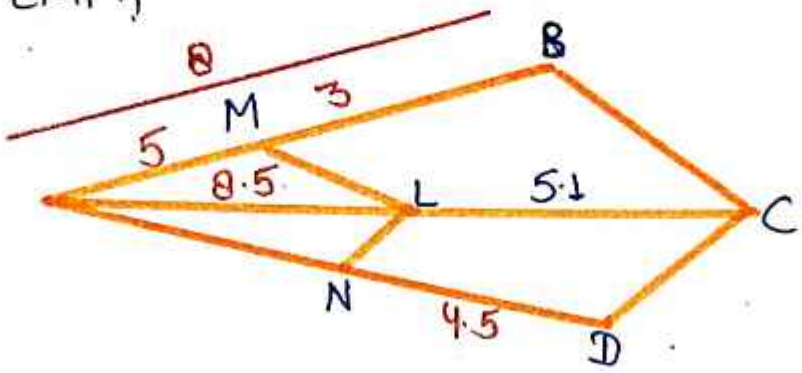
$$\frac{ar.\,\Delta AFE}{ar.\,\Delta ABC} = \left(\frac{AF}{AB}\right)^2 = \frac{1}{2}$$

$$\boxed{\frac{AF}{AB} = \frac{1}{\sqrt{2}}}$$

$$\frac{GD}{AG} = \frac{\sqrt{2}-1}{1}$$

Q) In the figure given below, LM ||CB and LN||CD. If AM = 5 cm, AB = 8 cm, ND = 4.5 cm and AL = 8.5 cm, then what will be the value of LC + AN?

नीचे दी गई आकृति में, LM||CB और LN||CD है। यदि AM = 5cm, AB = 8cm, ND = 4.5 cm और AL = 8.5 cm है, तो LC + AN का मान कितना होगा?



$$\frac{AM}{MB} = \frac{AL}{LC}$$

$$\frac{5}{3} = \frac{8.5}{LC} \quad 1.7$$

$$\boxed{LC = 5.1}$$

$$\frac{AL}{LC} = \frac{AN}{ND}$$

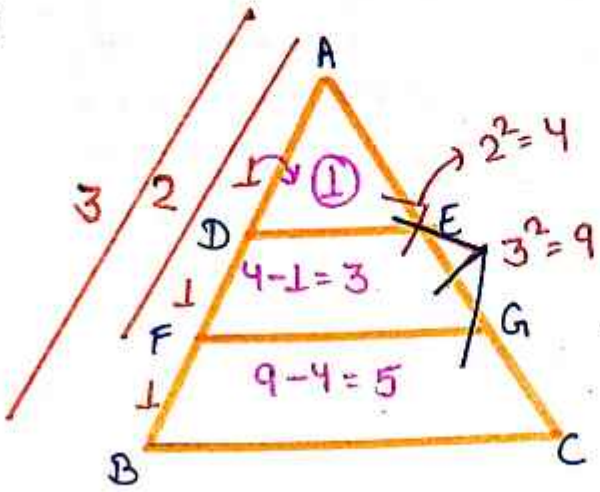
$$\frac{8.5}{5.1} = \frac{AN}{4.5} \quad 5,\ 3,\ 1.5$$

$$\boxed{AN = 7.5}$$

$$LC + AN$$
$$5.1 + 7.5$$
$$12.6$$

Q) The area of a triangle ABC is 63 square units. Two parallel lines DE, FG are drawn such that it divides AB and AC into three equal parts. What is the area of quadrilateral DEFG?

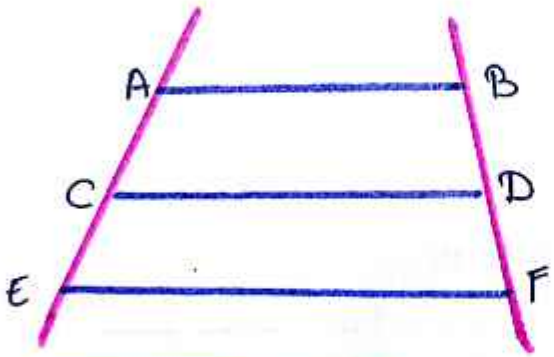
एक त्रिभुज ABC का क्षेत्रफल 63 वर्ग यूनिट है। दो समानान्तर रेखाएं DE, FG इस प्रकार खींची गई है कि यह AB तथा AC को तीन बराबर भागों में विभाजित करती है। चतुर्भुज DEFG का क्षेत्रफल कितना है?



AD : DF : FB
1 : 1 : 1

9 → 63
1 → 7
ar □ DEFG
3 → 3×7
21

LADDER THEOREM

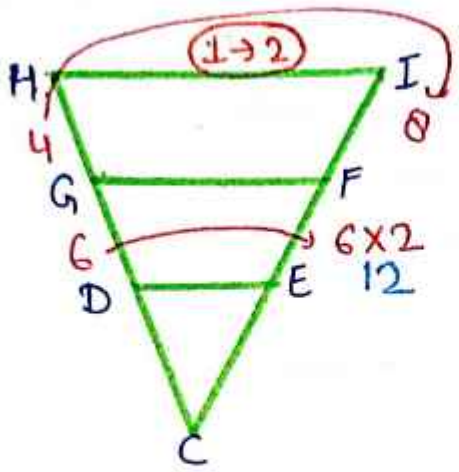


AB || CD || EF

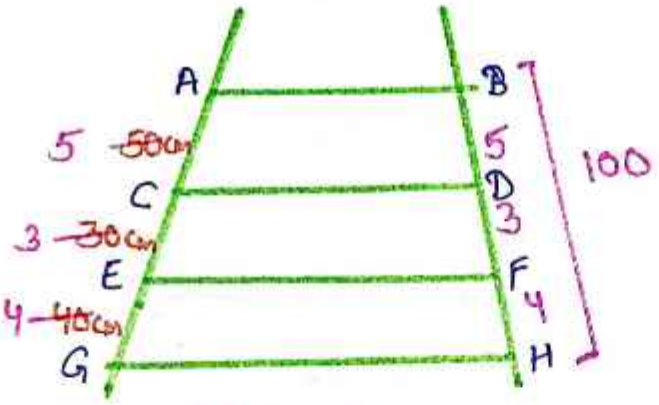
$$\frac{AC}{CE} = \frac{BD}{DF}$$

Q) HI, GF and DE are parallel lines, if DG = 6, GH = 4 and FI = 8, then EF = ?

HI, GF और DE समांतर रेखाएं हैं, यदि DG=6, GH=4 और FI=8, तो EF=?



Q) In the given figure AB ||CD|| EF ||GH and BH=100 cm. Find the value of DF:
दी गई आकृति में AB ||CD|| EF || GH तथा BH= 100 सेमी. है। DF का मान ज्ञात कीजिए:



12 → 100

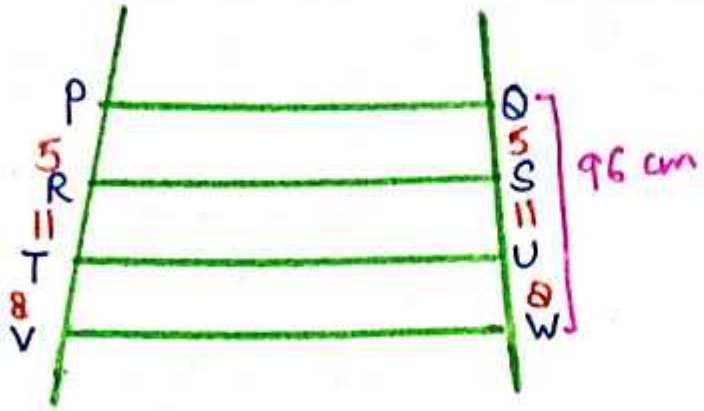
1 → 100/12

DF → 3 → 100/12 × 3 = 25/4

25cm

Q) In the figure given below PQ ||RS||TU||VW, PR= 5cm, RT= 11cm, TV= 8cm, QW= 96 cm then find QS?

दिए गए चित्र में, PQ||RS||TU||VW, भुजा PR= 5 सेमी. भुजा RT= 11 सेमी. भुजा TV= 8 सेमी. भुजा QW= 96 सेमी. है, तो भुजा QS का मान ज्ञात करें।



24 → 96 cm

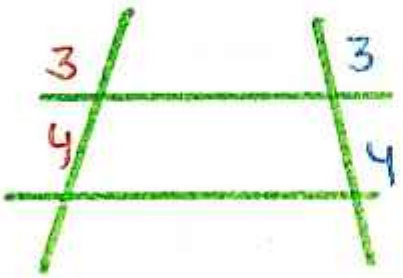
1 → 4 cm

QS → 5 → 5X4

20 cm

Q) When two transversals intersect three parallel lines, the ratio of intercepts formed by first transversal is 3:4, then find the ratio of intercepts formed by second transversal.

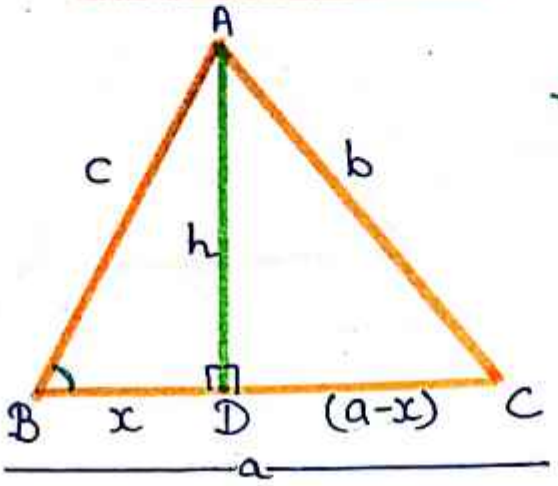
जब दो तिर्यक रेखाएँ, तीन समानांतर रेखाओं को प्रतिच्छेदित करती है, पहली तिर्यक रेखा द्वारा निर्मित अंतः खंडों का अनुपात 3:4 है, तो दूसरी तिर्यक रेखा द्वारा निर्मित अंतः खंडों का अनुपात ज्ञात कीजिए।



3 : 4

SINE AND COSINE RULE

* COSINE RULE *



Δ ABD

$x^2+h^2=c^2$

$h^2=c^2-x^2$

Δ ADC

$h^2=b^2-(a-x)^2$

$c^2-x^2=b^2-(a^2+x^2-2ax)$

$c^2-x^2=b^2-a^2-x^2+2ax$

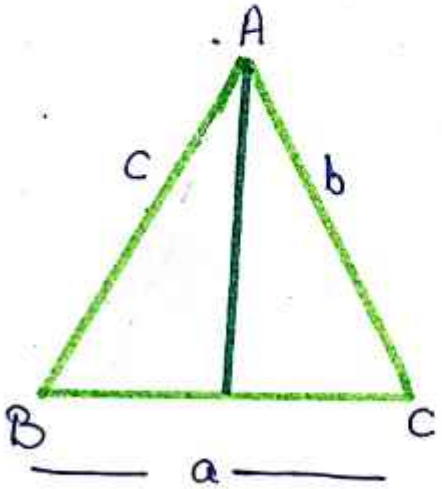
$2ax=a^2+c^2-b^2$

$2ac\cos B=a^2+c^2-b^2$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$\cos B=\frac{x}{c}$

$$x=c\cos B$$

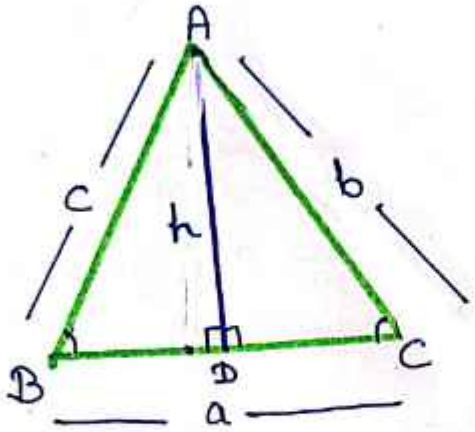


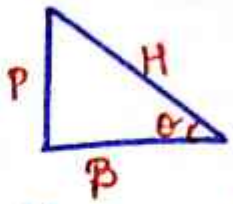
$$\cos A=\frac{c^2+b^2-a^2}{2cb}$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

* SINE RULE *





$$\left[\begin{array}{l} \cos\theta = \frac{B}{H} \\ \sin\theta = \frac{P}{H} \end{array}\right]$$

Δ ABD

$\sin B = \frac{h}{c}$

$\boxed{h = C \sin B}$

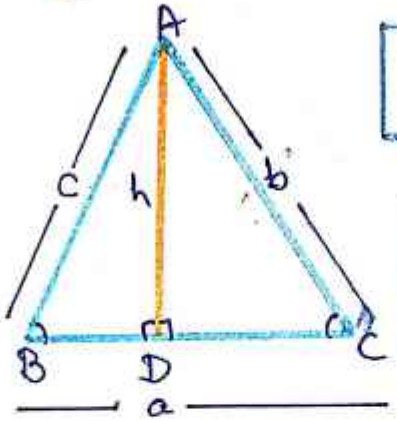
Δ ACB

$\sin C = \frac{h}{b}$

$\boxed{h = b \sin C}$

$$C \sin B = b \sin C$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$



$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

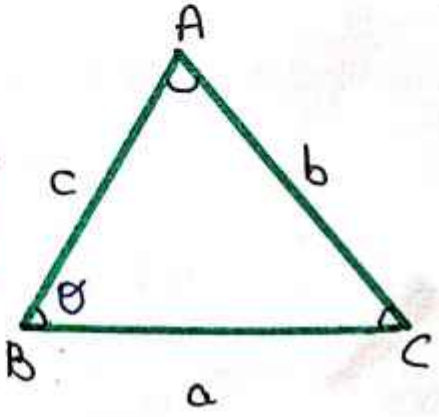
$\downarrow$

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

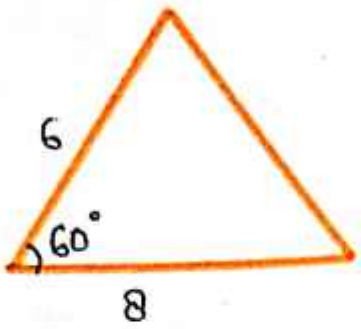
R- Radius of Circumcircle

परिवृत की त्रिज्या

$$\boxed{\text{Area of triangle} = \frac{1}{2} \times a \times c \times \sin\theta}$$



Ex:-



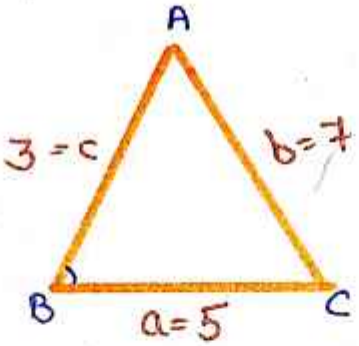
area of Δ = ?

$$\frac{1}{2} \times 6 \times 8 \times \sin 60^\circ$$

$$12 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$$

Q) In a triangle ABC, if the three sides are $a=5$, $b=7$ and $c=3$, what is angle B?

एक त्रिभुज ABC में, यदि तीन भुजाएँ हैं $a=5$, $b=7$ और $c=3$, कोण B क्या है?



Cosine Rule

$$\cos B = \frac{3^2+5^2-7^2}{2\times3\times5}$$

$$\frac{9+25-49}{30}$$

$$\frac{34-49}{30} = \frac{-15}{30} = -\frac{1}{2}$$

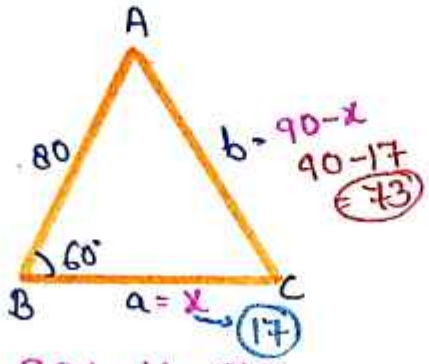
$\cos B = -\frac{1}{2} \Rightarrow$ angle $> 90^\circ$

$\cos 120^\circ = -\frac{1}{2}$

$\cos(90+\theta) = -\sin\theta$

Q) Side AB of triangle ABC is 80 cm long, whose perimeter is 170 cm. If $\angle ABC = 60°$, the shortest side of triangle ABC measures .... cm.

त्रिभुज ABC की भुजा AB 80 सेमी लंबी है, जिसका परिमाप 170 सेमी. है। यदि $\angle ABC = 60°$, त्रिभुज ABC के सबसे छोटी भुजा की माप सेमी मे क्या है?



$80 + a + b = 170$

$a + b = 90$

$$\cos 60° = \frac{80^2 + x^2 - (90-x)^2}{2 \times 80 \times x}$$

$$\frac{1}{2} = \frac{6400 + x^2 - (8100 + x^2 - 180x)}{2 \times 80 \times x}$$

$$80x = 6400 + x^2 - 8100 - x^2 + 180x$$

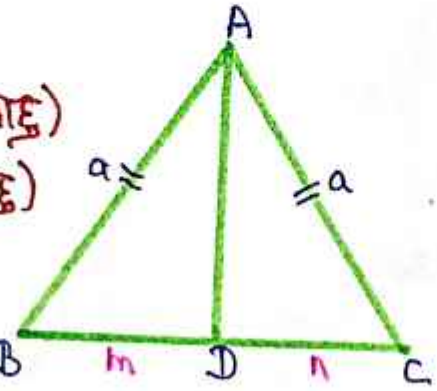
$$+100x = +1700$$

$$x = 17$$

STEWART THEOREM

Isosceles (समद्विबाहु)

Equilateral (समबाहु)



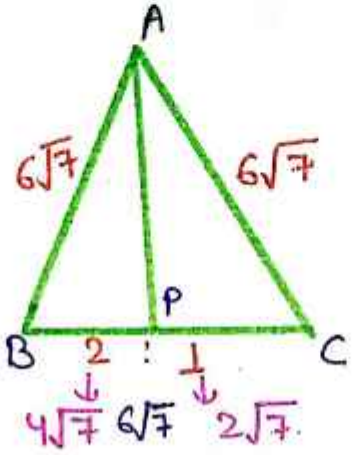
AD → any line on BC

BD : DC
m : n

$$AD^2 = a^2 - mn$$

Q) The side of an equilateral ΔABC is $6\sqrt{7}$ cm. P is a point on side BC such that BP : PC = 2 : 1. The length (in cm) of AP is:

समबाहु त्रिभुज ABC की भुजा $6\sqrt{7}$ सेमी है। BC पर बिंदु P इस प्रकार है कि BP : PC = 2 : 1 है। AP की लंबाई (सेमी में) ज्ञात करें।



$3 \rightarrow 6\sqrt{7}$

$1 \rightarrow 2\sqrt{7}$

Stewart theorem

$$AP^2 = (6\sqrt{7})^2 = 4\sqrt{7} \times 2\sqrt{7}$$

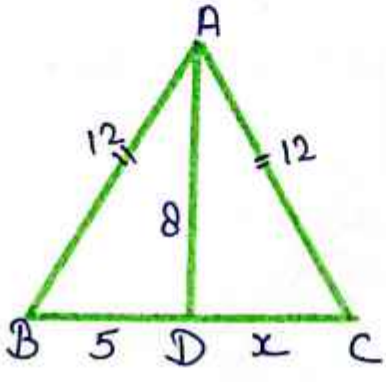
$$36 \times 7 - 56$$

$$252 - 56 = 196$$

$$AP = \sqrt{196} = 14$$

Q) In a triangle ABC, AB = AC. D is the point on BC. Find the length of CD if AB = 12 cm, AD = 8 cm, BD = 5 cm

किसी त्रिभुज ABC में, AB = AC है। D भुजा BC पर कोई बिंदु है। CD की लंबाई ज्ञात कीजिए यदि AB = 12 सेमी, AD = 8 सेमी, BD = 5 सेमी है।



$AD^2 = 12^2 - 5x$

$64 = 144 - 5x$

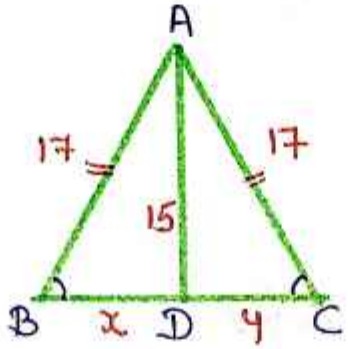
$5x = 144 - 64$

$5x = 80$

$x = \frac{80}{5}$ 16cm – CD

Q) In ΔABC, $AB = AC = 17$, D is a point on BC, $CD = 4$, $AD = 15$ then find the value of BD?

ΔABC में $AB = AC = 17$, BC पर बिंदु D स्थित है, जहाँ $CD = 4$, $AD = 15$ हो, तो BD का मान ज्ञात करें?



Stewart theorem

$AD^2 = 17^2 - 4x$

$15^2 = 17^2 - 4x$

$225 = 289 - 4x$

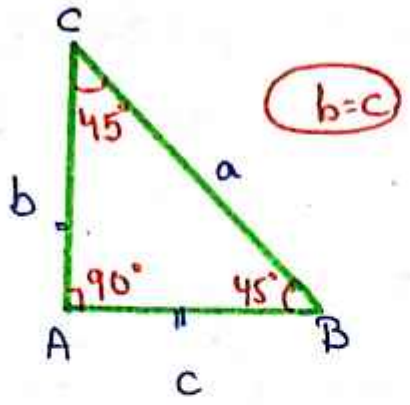
$4x = 289 - 225$

$4x = 64$

$x = \frac{64}{4}$ 16

Q) In right angle ΔABC, $AB = c$ cm, $AC = b$ cm and $CB = a$ cm. If $\angle A = 2\angle B$, then which of the following is true?

समकोण ΔABC में, $AB = c$ सेमी., $AC = b$ सेमी. और $CB = a$ सेमी. है। यदि $\angle A = 2\angle B$ है, तो निम्नलिखित में से कौन सा विकल्प सही है?



b=c

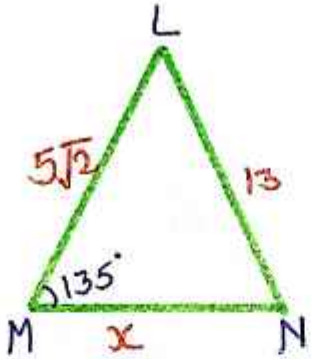
$a^2 = b^2 + c^2$

$a^2 = b^2 + c \times c$

$a^2 = b^2 + bc$

Q) In ΔLMN, $LM = 5\sqrt{2}$ cm, $LN = 13$ cm and $\angle LMN = 135°$. What is the length (in cm) of MN?

ΔLMN, $LM = 5\sqrt{2}$ सेमी, $LN = 13$ सेमी और $\angle LMN = 135°$, MN की लंबाई (सेमी में) क्या है?



$\cos 135° = \frac{(5\sqrt{2})^2 + x^2 - 13^2}{2 \times 5\sqrt{2} \times x}$

$\cos(135°) = \cos(90+45°)$

$= -\sin 45° = -\frac{1}{\sqrt{2}}$

$\frac{-1}{\sqrt{2}} = \frac{50 + x^2 - 169}{2 \times 5\sqrt{2} \times x}$

$10x = -119 + x^2$

$119 - 10x = x^2$

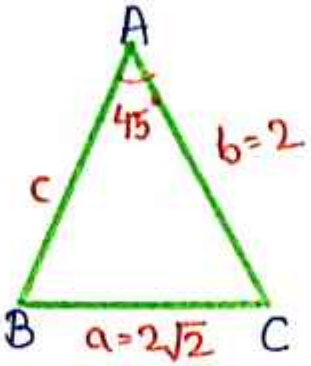
$x = 7$

$119 - 10 \times 7 = 7^2$

$119 - 70 = 49 = 49$

Q) In ΔABC, $b = 2$ & $\angle A = 45°$, $a = 2\sqrt{2}$, then find $\angle B$?

त्रिभुज ABC, $b = 2$, & $\angle A = 45°$, $a = 2\sqrt{2}$, $\angle B$ का मान ज्ञात करो।



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 45°}{2\sqrt{2}} = \frac{\sin B}{2}$$

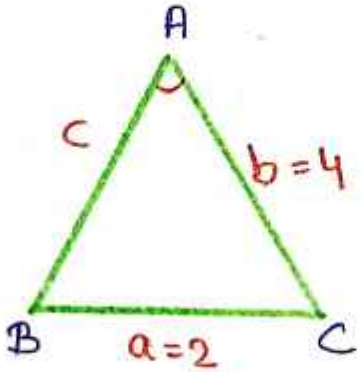
$$\frac{1}{\sqrt{2} \times 2\sqrt{2}} = \frac{\sin B}{2}$$

$$\sin B = \frac{1}{2}$$

$$B = 30°$$

Q) In a triangle ABC. If $a = 2$, $b = 4$ and $\sin A = \frac{1}{4}$, then what is angle B?

एक त्रिभुज ABC में यदि $a = 2$, $b = 4$ और $A = \frac{1}{4}$, तो कोण B क्या है?



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{1}{4 \times 2} = \frac{\sin B}{4}$$

$\sin B = \frac{1}{2}$ $\boxed{B = 30^\circ}$ $\pi = 180^\circ$

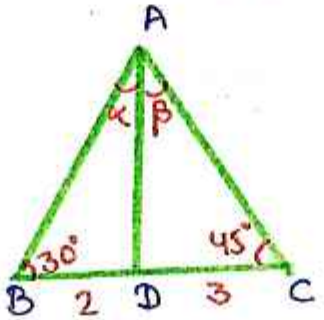
$\downarrow$ $\frac{180}{6}$ 30°

$\frac{\pi}{6}$

Q) In a ΔABC, AD divides BC in the ratio 2:3, $\angle B = 30^\circ$, $\angle C = 45^\circ$. Find $\frac{\sin \angle BAD}{\sin \angle CAD}$?

ΔABC में, AD भुजा BC को 2:3 अनुपात में विभाजित करता है और $\angle B = 30^\circ$, $\angle C = 45^\circ$ है। तो $\frac{\sin \angle BAD}{\sin \angle CAD}$ ज्ञात कीजिए

$\frac{\sin \alpha}{\sin \beta}$



ΔABD

$\frac{\sin \alpha}{2} = \frac{\sin 30}{AD}$

$AD \sin \alpha = 2 \times \frac{1}{2}$ ①

ΔACD

$\frac{\sin \beta}{3} = \frac{\sin 45^\circ}{AD}$

$AD \sin \beta = 3 \times \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

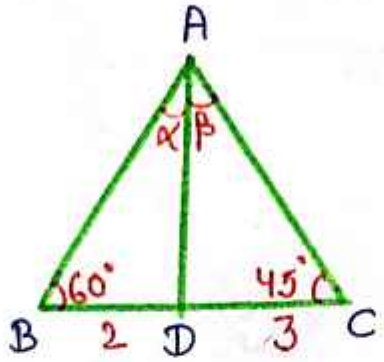
$\frac{AD \sin \alpha}{AD \sin \beta} = \frac{1}{3} \times \sqrt{2}$

$\frac{\sqrt{2}}{3}$

Q) In triangle ABC, AD divides BC in 2:3 if $\angle B = 60^\circ$ and $\angle C = 45^\circ$ then find $\frac{\sin \angle BAD}{\sin \angle CAD}$.

त्रिभुज ABC में, AD, BC को 2:3 में विभाजित करती है यदि $\angle B = 60^\circ$ और $\angle C = 45^\circ$ तो $\frac{\sin \angle BAD}{\sin \angle CAD}$ ज्ञात करे।

$\frac{\sin \alpha}{\sin \beta}$



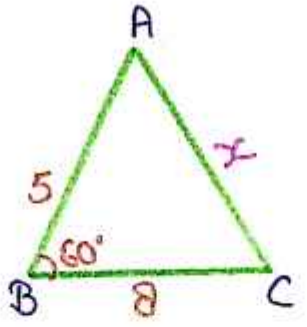
$$\frac{\sin\alpha}{2} = \frac{\sin 60}{AD} \qquad \frac{\sin\beta}{3} = \frac{\sin 45}{AD}$$

$$AD\sin\alpha = \cancel{2}\times\frac{\sqrt{3}}{\cancel{2}} = \sqrt{3} \qquad AD\sin\beta = 3\times\frac{1}{\sqrt{2}}$$

$$\frac{\cancel{AD}\ \sin\alpha}{\cancel{AD}\ \sin\beta} = \frac{\sqrt{3}}{3}\times\sqrt{2} = \frac{\sqrt{2}}{\sqrt{3}}$$

Q) In ΔABC, $AB = 5$ cm, $BC = 8$ cm and $\angle ABC = 60°$, then AC equal to:

ΔABC में $AB = 5$ सेमी, $BC = 8$ सेमी और $\angle ABC = 60°$ हो, तो AC बराबर है।



$$\cos 60° = \frac{5^2+8^2-x^2}{2\times5\times8}$$

$$\frac{1}{\cancel{2}} = \frac{25+64-x^2}{\cancel{2}\times40}$$

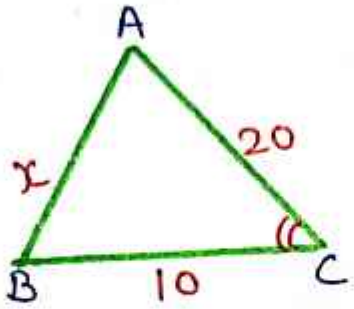
$$40 = 89 - x^2$$

$$x = 89 - 40 = 49$$

$$x = \sqrt{49} = 7$$

Q) In ΔABC, $AC = 20$, $BC = 10$ and area of $\Delta ABC = 80$. Find the value of B?

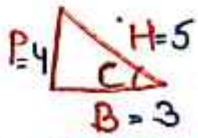
ΔABC में $AC = 20$, $BC = 10$ तथा ΔABC का क्षेत्रफल 80 है, तो AB का मान ज्ञात करो?



area = 80

$80 \times \frac{1}{2} \times 10 \times 20 \sin C$

$\sin = \frac{8}{10} \frac{4}{5}$



$\sin C = \frac{P}{H}$

$\cos C = \frac{3}{5}$

$\cos C = \frac{10^2 + 20^2 - x^2}{2 \times 10 \times 20}$

$\frac{3}{5} = \frac{100 + 400 - x^2}{400}$

80

$240 = 500 - x^2$

$x^2 = 500 - 240 = 260$

$x = \sqrt{260}$

1. In triangle ABC, AB=12" " cm and AC= 10 cm , and $\angle BAC = 60^\circ$ What is the length (in cm) of side BC ?

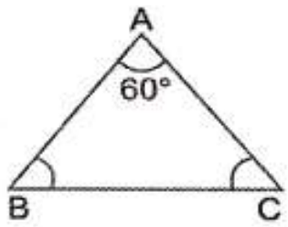
त्रिभुज ABC में $AB = 12$ cm और $AC = 10$ cm , और $\angle BAC = 60^\circ$ है। भुजा BC की लंबाई का मान क्या है?

(a) 10 सेमी.

(b) 7.13 सेमी.

(c) 13.20 सेमी.

(d) 11.13 सेमी.



2. In a triangle $ABC, \angle B = 30^\circ$ and $\angle C = 45^\circ$. If $BC = 50$ cm then find the length of AB ?

किसी त्रिभुज ABC में $\angle B = 30^\circ$ और $\angle C = 45^\circ$ है। यदि $BC = 50$ सेमी. है तो AB की लंबाई ज्ञात करें।

(a) $\frac{50}{\sqrt{3}+1}$

(b)) $50(\sqrt{3}-1)$

(c) $\frac{100}{(\sqrt{3}-1)}$

(d) $100(\sqrt{3}-1)$

3. Sides of a triangle are $(K^2+K+1), (2K+1)$ and (K^2-1) then find the greatest value of angle in triangle?

त्रिभुज की भुजाएँ $(K^2+K+1), (2K+1)$ और (K^2-1) है। त्रिभुज के सबसे बड़े कोण का मान बताइए।

(a) 90°

(b) 75°

(c) 120°

(d) 105°

4. In a triangle ABC if $a = 3, b = 4$ and $\sin A = 3/4$, then what is angle B equal to?

त्रिभुज ABC में, यदि $a = 3, b = 4$ और $\sin A = 3/4$ है, तो कोण B किसंके बराबर है?

(a) $\pi/4$

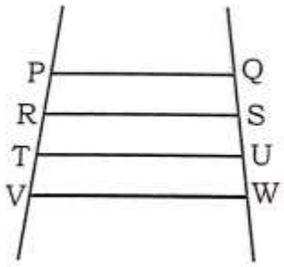
(b) $\pi/2$

(c) $\pi/3$

(d) $\pi/6$

5. In the fig given below $PQ \parallel RS \parallel TU \parallel VW, PR = 20$ cm, $RT = 44$ cm, $TV = 32$ cm, $QW = 84$ cm then find QS ?

दिए गए चित्र में, $PQ \parallel RS \parallel TU \parallel VW$, भुजा $PR = 20$ सेमी. भुजा $RT = 44$ सेमी. भुजा $TV = 32$ सेमी. भुजा $QW = 84$ संमी. है, तो भुजा QS का मान ज्ञात करें।

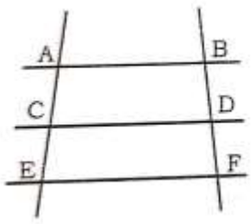


(a) 15 cm
(b) 17.5 cm
(c) 22.5 cm
(d) 12.5 cm

6. In the given fig below $AB \parallel CD \parallel EF$, if $AB = 29\ cm, EF = 57\ cm, AC = \frac{3}{4}CE$ and BD is x cm less than DF then find CD = ?

नीचे दिए गए चित्र में $AB \parallel CD \parallel EF$,

यदि भुजा $AB = 29$ सेमी., भुजा $EF = 57$ सेमी., $AC = \frac{3}{4}CE$ और भुजा BD , भुजा DF से x सेमी. कम है, तो भुजा CD का मान क्या होगा?

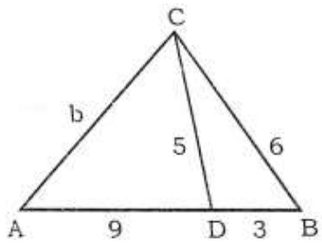


(a) 41 cm
(b) 43 cm
(c) 45 cm
(d) 40.5 cm

7. In the given figure find the length of AC ?

दी गई आकृति में, AC की लंबाई ज्ञात कीजिए?

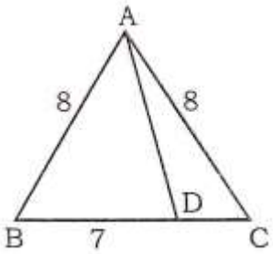
(A) 10
(B) 12
(C) 8
(D) 13



8. In the given fig. if $AB = AC = 8\ cm, BC = 11\ cm, BD = 7\ cm$ then AD = ?

दिए गए चित्र में यदि $AB = AC = 8$ सेमी., $BC = 11$ सेमी., $BD = 7$ सेमी. तब AD = ?

(a) 6 cm
(b) 5.5 cm
(c) 4 cm
(d) 7 cm



9. In △PQR, S and T are the mid-points of PQ and PR respectively. If $\angle QPR = 75°$ and $\angle PRQ = 40°$, then ∠TSQ is :

△ PQR में, S और T क्रमशः PQ और PR के मध्य-बिंदु हैं, यदि $\angle QPR = 75°$ और $\angle PRQ = 40°$, तो $\angle TSQ$ है :

(a) 135°
(b) 120°
(c) 105°
(d) 115°

10. In $\triangle$ABC, a line is drawn parallel to BC which cuts the sides AB and AC at points S and T, where AB=8.3 cm, BC= 16.6 cm and BS=5.3 cm. Find the length of ST (in cm).

$\triangle$ ABC में, BC के समानांतर एक रेखा खींची गई है, जो भुजाओं AB और AC को बिंदु S और T पर काटती है, जहाँ $AB = 8.3$ cm, $BC = 16.6$ cm और $BS = 5.3$ cm है। ST की लंबाई (cm में) ज्ञात कीजिए।

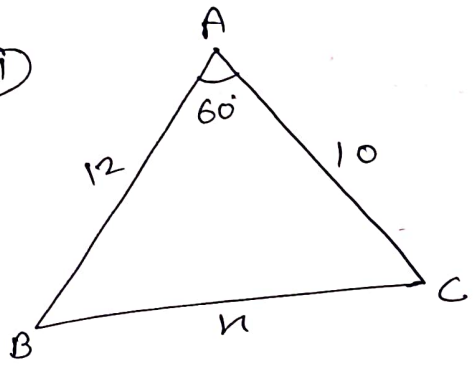
(a) 6
(b) 12
(c) 24
(d) 18

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
D	B	C	B	B	A	A	A	D	A

worksheet solution

(1)



$$\cos 60^\circ = \frac{12^2 + 10^2 - x^2}{2 \times 12 \times 10}$$

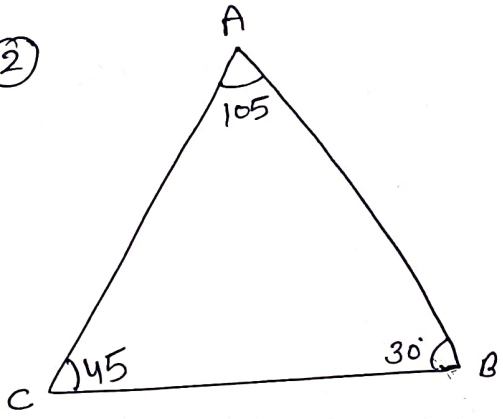
$$\frac{1}{2} = \frac{244 - x^2}{240}$$

$$240 = 2(244 - x^2)$$

$$x^2 = 244 - 120$$

$$x = 11.13 \text{ cm Ans}$$

(2)



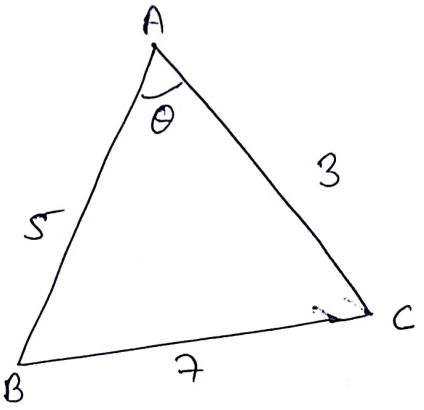
$$\frac{AB}{50} = \frac{\sin 45}{\sin 105}$$

$$\frac{AB}{50} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$AB = \frac{100}{(\sqrt{3}+1) \times 2} = 50(\sqrt{3}+1) \text{ Ans}$$

(3) Put $K=2$

sides = 7, 5, 3

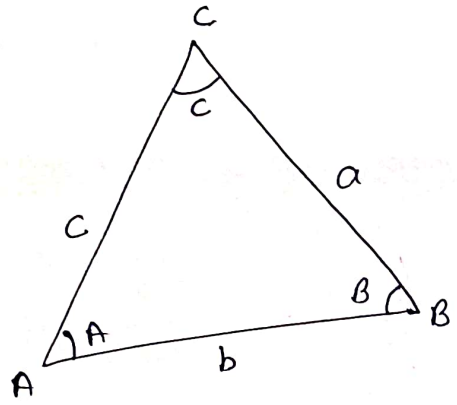


$$\cos\theta = \frac{25+9-49}{2\times5\times3} = \frac{-15}{30} = \frac{-1}{2}$$

$\cos 120^\circ$

$\theta = 120^\circ$ Ans

④



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{3/4}{3} = \frac{\sin B}{4}$$

$$\sin B = 1$$

$$B = 90^\circ$$

$$\frac{\pi}{2} \text{ Ans}$$

⑤ $PR : RT : TV = 20 : 44 : 32$

$5 : 11 : 8$

$QS : SU : UW = 5 : 11 : 8$

$$QS = 84 \times \frac{5}{24}$$

17.5 cm Ans

⑥ $AC : CE = 3 : 4$

$$CD = \frac{29 \times 4 + 57 \times 3}{4+3}$$

$$\frac{287}{7} = 41 \text{ cm Ans}$$

(7) $b^2 \times \cancel{3} + 6^2 \times \cancel{9}_{3} = 5^2 \times \cancel{12}_{4} + 9 \times \cancel{3} \times 12$

$b^2 + 108 = 100 + 108$

$b^2 = 100$

$b = 10$ cm Ans

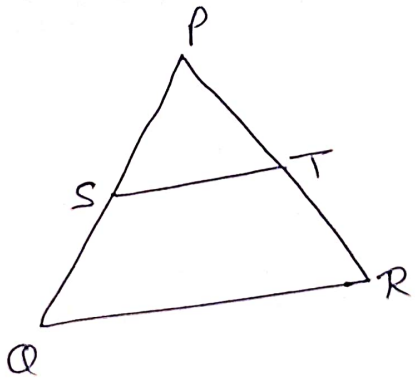
(8)

$AD^2 = 8^2 - 7 \times 4$

$AD^2 = 36$

$AD = 6$ cm Ans

(9)



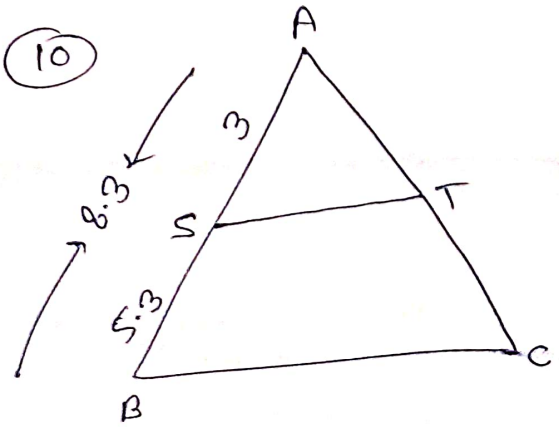
$\angle PQR = 180 - 75 - 40$

$\Rightarrow 65^\circ$

$\angle PQR = \angle PST = 65^\circ$

$\angle QST = 180^\circ - 65^\circ = 115^\circ$ Ans

(10)



$\frac{AS}{AB} = \frac{sf}{BC}$

$\frac{3}{8.3} = \frac{sf}{16.6}$

$sf = \frac{16.6 \times 3}{8.3}$

6 cm Ans