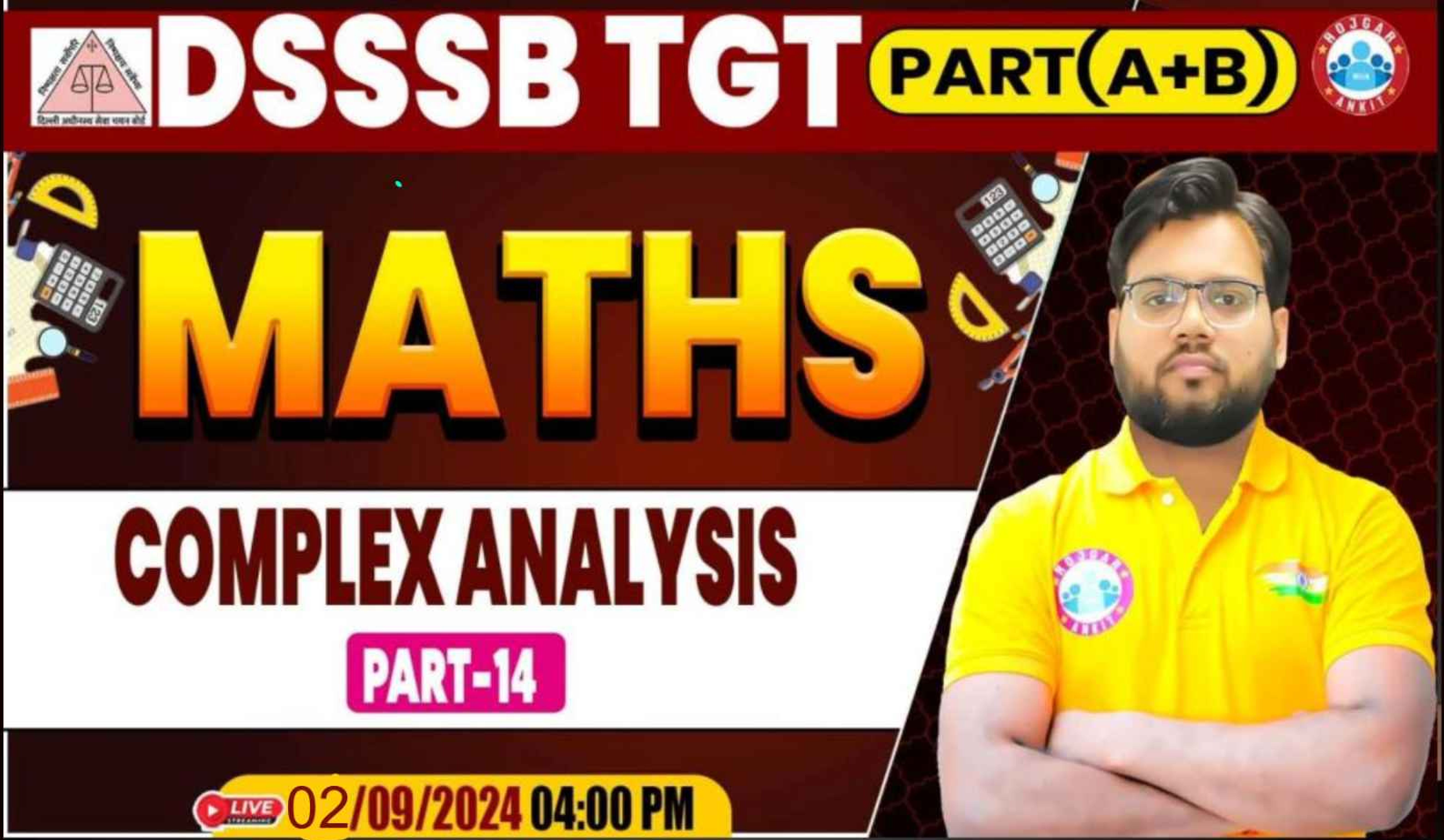
DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-14
LIVE STREAMING
02/09/2024 04:00 PM

Complex Function

Complex Function: - A function $f : D \subseteq \mathbb{C} \rightarrow \mathbb{C}$ is said to be a Complex Function

Complex Analysis

Trigonometric Function

Some Results: -

$$\sin^2 z + \cos^2 z = 1; \quad 1 + \tan^2 z = \sec^2 z$$

$$1 + \cot^2 z = \operatorname{cosec}^2 z; \quad \sin(-z) = -\sin z$$

$$\cos(-z) = \cos z; \quad \tan(-z) = -\tan z$$

Hyperbolic Function: -

$$\sinh z = \frac{e^z - e^{-z}}{2}; \cosh z = \frac{e^z + e^{-z}}{2}$$

$$\operatorname{sech} z = \frac{1}{\cosh z} = \frac{2}{e^z + e^{-z}}; \operatorname{cosech} z = \frac{1}{\sinh z} = \frac{2}{e^z - e^{-z}}$$

$$\tanh z = \frac{\sinh z}{\cosh z} = \frac{e^z - e^{-z}}{e^{-} + e^{-z}}; \coth z = \frac{\cosh z}{\sinh z} = \frac{e^z + e^{-z}}{e^z - e^{-z}}$$

$$\sinh(z_1 \pm z_2) = \sinh z_1 \cdot \cosh z_2 \pm \cosh z_1 \cdot \sinh z_2$$

$$\sin(iz) = i\sinh z; \cos(iz) = \cosh z$$

$$\tan(iz) = i\tanh z; \sinh(iz) = i\sin z$$

$$\cosh(iz) = \cos z; \tanh(iz) = i\tan z$$

$$\cosh^2 z - \sinh^2 z = 1$$

$$1 - \tanh^2 z = \operatorname{sech}^2 z$$

$$\coth^2 z - 1 = \operatorname{cosech}^2 z$$

Q. The value of $\cosh^2 z - \sinh^2 z$

$\cosh^2 z - \sinh^2 z$ **का मान-**

(a) $\cosh(2z)$

(b) 1

(c) $\sinh(2z)$

(d) 0

Q. The Real part of $\sin(x+iy)$ –

$\sin(x+iy)$ **का वास्तविक भाग है**

(a) $\sin x$

(b) $\cos y$

(c) $\sin x\cos hy$

(d) $\sin x\sin hy$

Q. What will be the imaginary/part of $\cosh(\alpha + i\beta)$ **?**

$\cosh(\alpha + i\beta)$ **का काल्पनिक भाग होगा?**

(a) $\cosh \alpha \cos \beta$

(b) $\sinh \alpha \sin \beta$

(c) $-\sin h\alpha \sin \beta$

(d) $-\cos h\alpha \cos \beta$

Logarithmic Function:

Q. $\log(1+i)$ **is equal to :/log** $(1+i)$ **बराबर है-**

(a) $\frac{1}{2}\log\ 2$

(b) $\frac{\pi}{4}+\frac{1}{2}\log\ 2$

(c) $\frac{1}{2}\log\ 2+\mathrm{i}\frac{\pi}{4}$

(d) None of these

Q. Find out the all values of Z from this equation:

$e^z = 1 + \sqrt{3}i.$

a. $\ln(2) - (\pi \div 2 + 2\pi n)i; n \in Z$

b. $\ln(2) - (\pi \div 3 + \pi n)i; n \in Z$

c. $\ln(2) + (\pi \div 2 + \pi n)i; n \in Z$

d. $\ln(2) + (\pi \div 3 + 2\pi n)i; n \in Z$

Inverse Trigonometric Function:

$$\sin^{-1} z = \frac{1}{i}\ln\left(iz + \sqrt{1 - z^2}\right)$$

$$\operatorname{cosec}^{-1} z = \frac{1}{i}\ln\left(\frac{i+\sqrt{z^2-1}}{z}\right)$$

$$\cos^{-1} z = \frac{1}{i}\ln\left(z + \sqrt{z^2 - 1}\right)$$

$$\sec^{-1} z = \frac{1}{i}\ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$

$$\tan^{-1} z = \frac{1}{2}\ln\left(\frac{1+iz}{1-iz}\right)$$

$$\cot^{-1} z = \frac{1}{2i}\ln\left(\frac{z+i}{z-i}\right)$$

Inverse Hyperbolic Function:

$$\sinh^{-1} z = \ln\left(z + \sqrt{z^2 + 1}\right)$$

$$\operatorname{cosech}^{-1} z = \ln\left(\frac{1+\sqrt{z^2-1}}{z}\right)$$

$$\cosh^{-1} z = \ln\left(z + \sqrt{z^2 - 1}\right)$$

$$\operatorname{sech}^{-1} z = \ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$

$$\tanh^{-1} z = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$$

$$\coth^{-1} z = \frac{1}{2}\ln\left(\frac{z+1}{z-1}\right)$$

Q. $\operatorname{sech}^{-1}\left(\frac{1}{2}\right)$ **is equal to :**

$\operatorname{sech}^{-1}\left(\frac{1}{2}\right)$ **बराबर है:**

(a) $\log(\sqrt{3} \pm \sqrt{2})$

(b) $\log(\sqrt{3} \pm 1)$

(c) $\log(2 \pm \sqrt{3})$

(d) इनमें से कोई नहीं

Principal Logarithm Function:

Q. The principal value of $\log\left(i^{1/4}\right)$ **is**

(a) πi

(b) $\frac{\pi}{2} i$

(c) $\frac{\pi}{4} i$

(d) $\frac{\pi}{8} i$

Periods of Function: -

$$\sin az = \frac{2\pi}{|a|}$$

$$\cos az = \frac{2\pi}{|a|}$$

$$\tan az = \frac{\pi}{|a|}$$

$$\operatorname{cosec} az = \frac{2\pi}{|a|}$$

$$\sec az = \frac{2\pi}{|a|}$$

$$\cot az = \frac{\pi}{|a|}$$

$\sin 5z$ का period = ?

$$\Rightarrow \frac{2\pi}{5}$$

period of $\sin(-5z) = ?$

$$= \frac{2\pi}{|-5|} \Rightarrow \frac{2\pi}{5}$$

Periods of Hyperbolic fu^n

$$\sinh z = 2\pi i$$

$$\cosh z = 2\pi i$$

$$\tanh z = \pi i$$

$$\operatorname{cosech} z = 2\pi i$$

$$\operatorname{sech} z = 2\pi i$$

$$\coth z = \pi i$$

Q. The solution of the equation $\cos z = 2$, **whe**

$z = x + iy$ **is**

समीकरण $\cos z = 2$ **जहाँ** $z = x + iy$ **का हल है**

(a) $z = 2n\pi \mp i\log(2+\sqrt{3})$

(b) $z = 2n\pi \pm i\log(1+\sqrt{3})$

(c) $z = 2n\pi \pm i\log(2-\sqrt{3})$

(d) $z = 2n\pi \pm i\log(1-\sqrt{3})$

$z = 2n\pi + i\log(2+\sqrt{3})$

$\cos(x+iy) = 2$

$\cos x \cdot \cos iy - \sin x \cdot \sin iy = 2$

$\underbrace{\cos x \cdot \cosh y}_{\text{Real}} - i\sin x \cdot \sinh y = \underbrace{2}_{\text{Real}} + 0i$

$\cos x \cdot \cosh y = 2$

$1 \cdot \cosh y = 2$

$y = \cosh^{-1} 2$

$y = \log(2+\sqrt{3})$

$-\sin x \cdot \sinh y = 0 \rightarrow$ Not possible zero

$x = 0 \quad y = 0$

$x = 0, \pi, 2n\pi$

Complex Exponent : $z^{\alpha} = e^{\alpha \log z}; z \neq 0$

$z^{\alpha} = e^{\alpha \log z}$

$\Rightarrow e^{\alpha[\log|z| + i(\arg z + \underline{2n\pi})]}$

Principal values of z^{α}:

$z^{\alpha} = e^{\alpha \log z}$

$z = (x + iy)$

$\Rightarrow e^{\alpha[\log|z| + i \arg z]}$

Q. i^i Find Principal Value?

(a) $e^{\frac{\pi}{2}}$

(b) $e^{-\frac{\pi}{2}}$ ✓

(c) $e^{-\frac{i\pi}{2}}$

(d) None of these.

$$z^\alpha = e^{\alpha \log z}$$

$$i^i$$

$$\log z = \log|z| + i \arg z$$

$$z^\alpha = (0+i)^i = e^{i \log(0+i)}$$

$$\Rightarrow e^{i\left[\log|0+i| + i\tan^{-1}\left|\frac{1}{0}\right|\right]}$$

$$\Rightarrow e^{i\left[\log 1 + i\frac{\pi}{2}\right]} \Rightarrow e^{i\left[i\frac{\pi}{2}\right]} \Rightarrow e^{-\frac{\pi}{2}}$$

Q. What will be the Real part of the p

value of 2^{-i}? $e^{i\theta} = \cos\theta + i\sin\theta$

2^{-i} के मुख्य मान का वास्तविक भाग होगा?

(a) $\sin(\log 2)$

(b) $\cos\left(\frac{1}{\log 2}\right)$

(c) $\boxed{\cos(\log 2)}$

(d) $\cos(\sin 2)$

2^{-i}

$z^{\alpha} \Rightarrow (2+0i)^{-i}$

$|2+0i|$

$\sqrt{4+0}$

$\Rightarrow e^{-i\log(2+0i)}$

$\Rightarrow e^{-i[\log|2+0i| + i\arg(2+0i)]}$

$\Rightarrow e^{-i[\log 2 + i.0]}$

$\Rightarrow e^{-i\log 2}$

$\Rightarrow \underbrace{\cos(\log 2)}_{\text{Real}} - i\underbrace{\sin(\log 2)}_{\text{Im}}$

$z^{\alpha} = e^{\alpha\log z}$

$= e^{\alpha[\log|z| + i\arg z]}$

$\arg(2+0i)$

$\tan^{-1}\left|\frac{y}{x}\right|$

$\tan^{-1}\left|\frac{0}{2}\right|$

$\Rightarrow \boxed{e^{-i\theta} = \cos\theta - i\sin\theta}$

Q. Find the value of $(1+i)^i$?

(a) $e^{\left[\ln 2^{\frac{1}{2}} - i\left(\frac{\pi}{4}+2n\pi\right)\right]}$

(b) $e^{i\left[\ln 2^{\frac{1}{2}} - i\left(\frac{\pi}{4}2n\pi\right)\right]}$

(c) $e^{i\left[\ln 2^{1/2} + i\left(\frac{\pi}{2}+n\pi\right)\right]}$

(d) $e^{i\left[\ln 2^{1/2} + i\left(\frac{\pi}{4}+2n\pi\right)\right]}$

$z^\alpha = e^{\alpha \log z}$

$z^\alpha = (1+i)^i$

$= e^{i \log(1+i)}$

$\log|1+i|$

$\sqrt{1^2+1^2} \Rightarrow \sqrt{2}$

$\Rightarrow e^{i\left[\log\sqrt{2} + i\left(\frac{\pi}{4}+2n\pi\right)\right]}$

$\Rightarrow e^{i\left[\ln 2^{\frac{1}{2}} + i\left(\frac{\pi}{4}+2n\pi\right)\right]}$