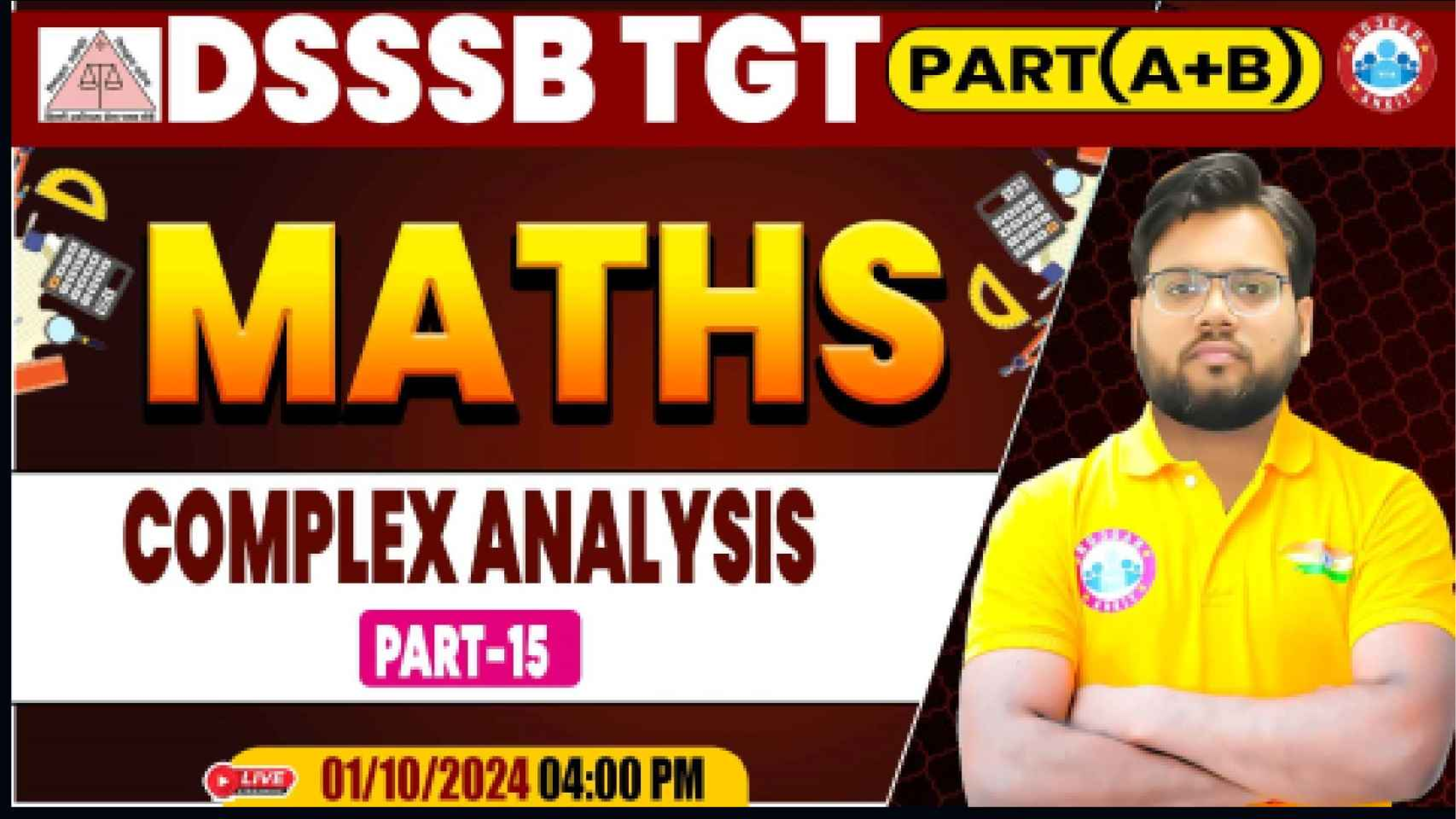
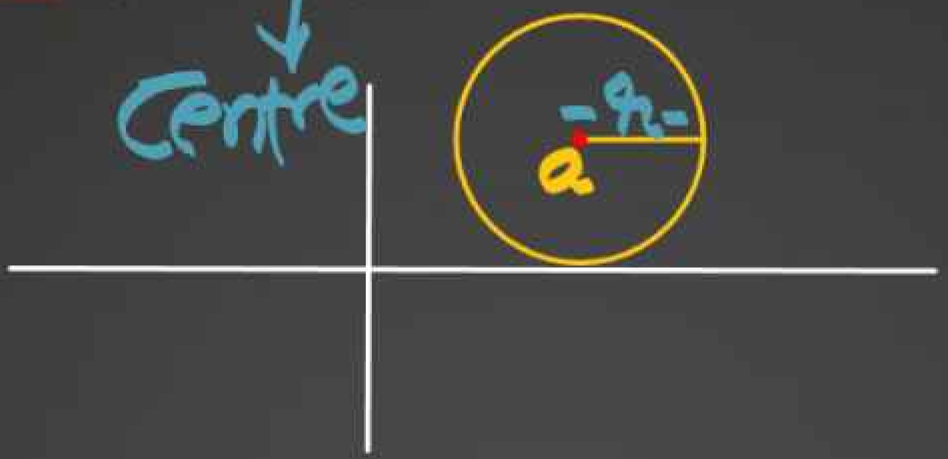
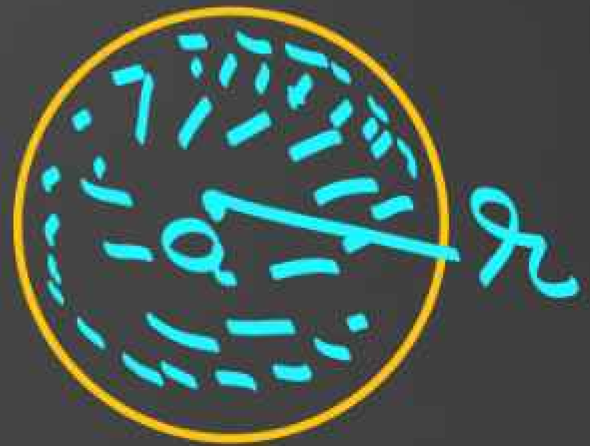

DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-15
LIVE
01/10/2024 04:00 PM

Circle: $|z - a| = r$ → Radius



Open disc or Open ball:

$$B(a, r) = \{z \in \mathbb{C} : |z - a| < r\}$$

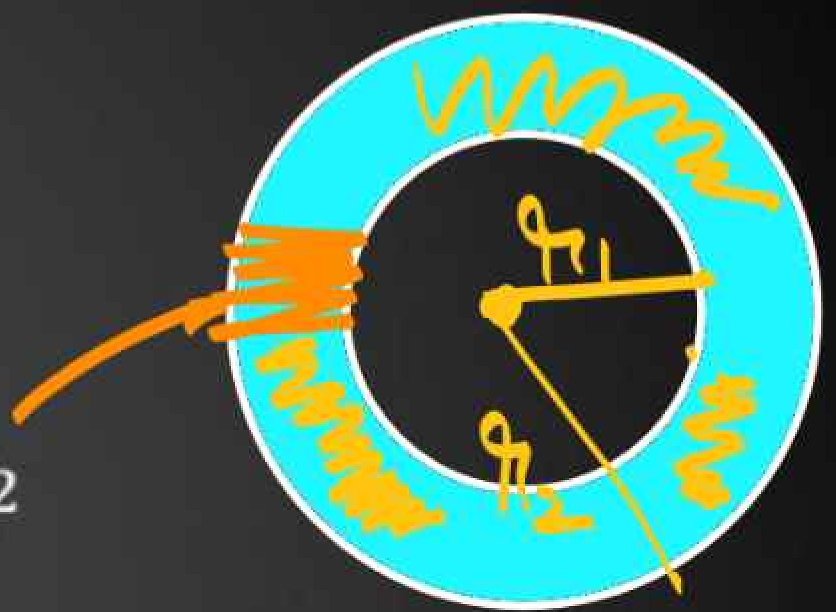


Closed disc or Closed ball:

$$B[a, r] = \{z \in \mathbb{C} : |z - a| \leq r\}$$

Annulus:

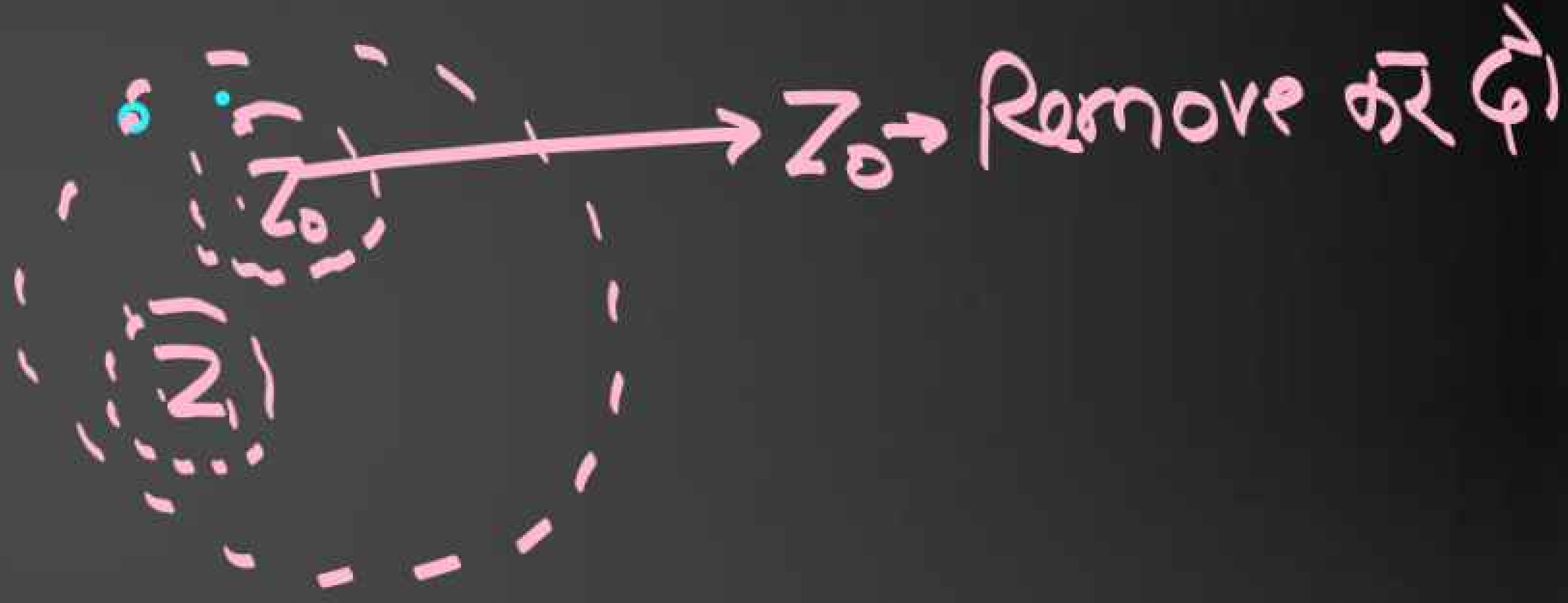
open annulus: $r_1 < |z - a| < r_2$



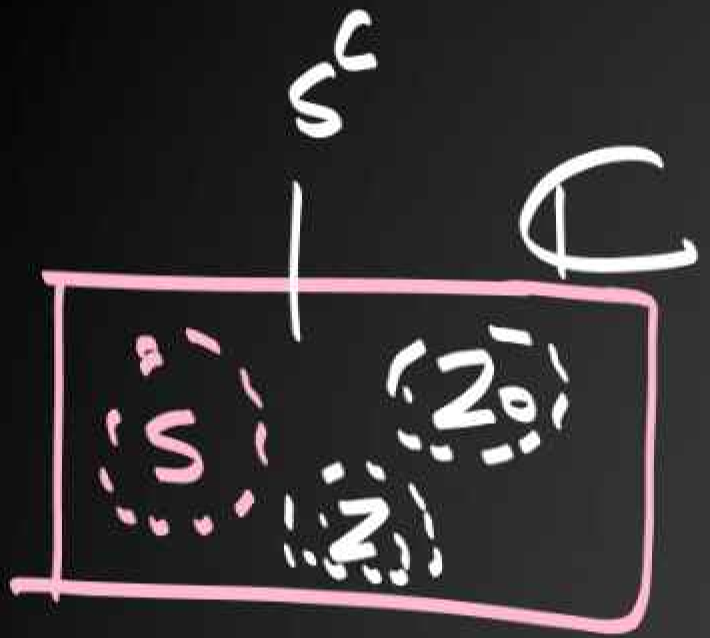
Closed annulus: $r_1 \le |z - z_0| \le r_2$

Neighborhood of a point: **- A δ - nbd. of a point z_0 in the complex plane is the set of all points z which lie in the open ball $|z - z_0| < \delta$.**

Deleted neighbourhood: - The deleted nbd. of a Complex no. z_0 corresponding to a given real positive no. δ, is the set of all z points for which $0 < |z - z_0| < \delta$.



Interior Point: - let $S \subseteq \mathbb{C}$. Then, a point $Z_0 \in S$ is said to be interior point of S if there exist at-least one nbd. of z_0 which lies entirely inside of the set S.

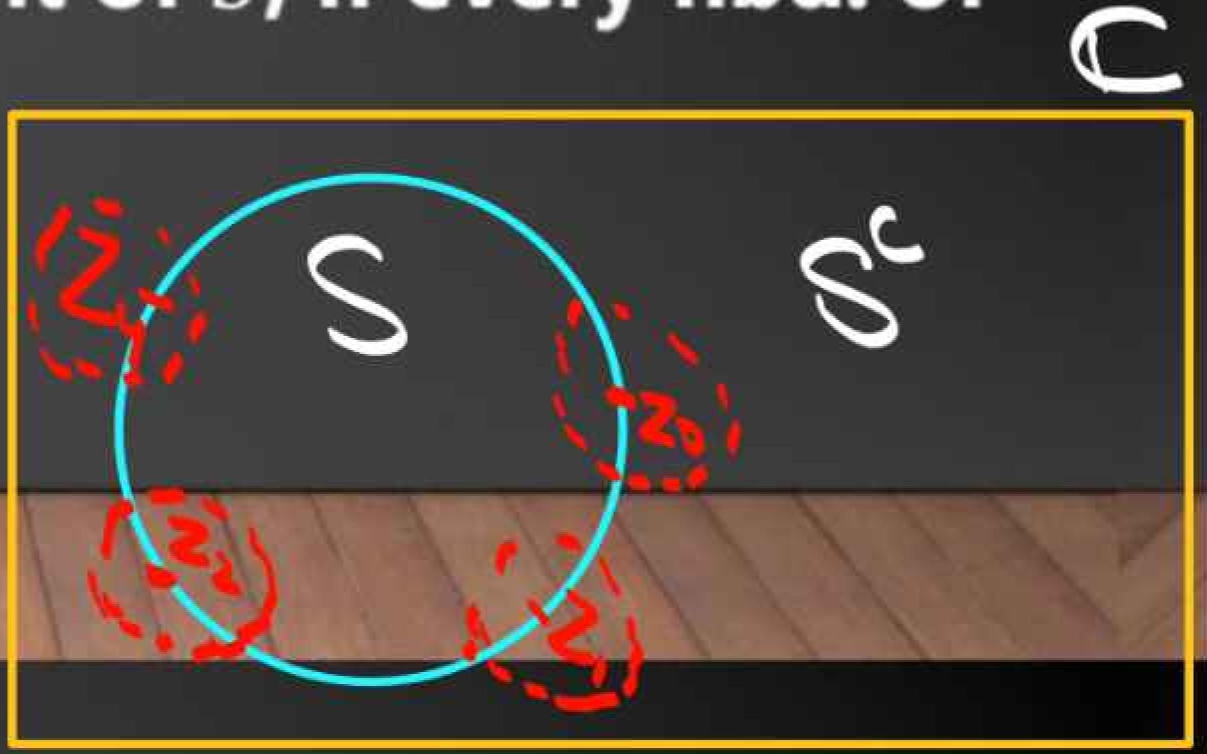


Exterior Point: - Let $S \subseteq \mathbb{C}$. Then, z_0 is said to be an exterior point of S if there exist at-least one nbd. of z_0 which lies entirely outside of the set S.

Boundary point: - A set $S \subseteq \mathbb{C}$, then a point z_0 is said to be a boundary point of S, if every nbd. of z_0 intersect with S and S^c.

$$S \cup S^c = \mathbb{C}$$

$$S \cap S^c = \phi$$



Limit point / Accumulation point/ Cluster points:

- **Let $S \subseteq \mathbb{C}$. Then, a point z_0 is said to be limit point of S, if every nbd. of z_0 Contains infinite points of S.**

Open set: - **A set $S \subseteq \mathbb{C}$ is said to be open if it is a nbd. of each of its pts.**

Closed set: - A set $S \subseteq \mathbb{C}$ is said to be closed set if it contains all of its limit points.

Bounded Set: - A set $S \subseteq \mathbb{C}$ is called bounded if there exist a positive real number M, such that $|z| < M \forall z \in S$; Otherwise, the set S is called unbounded.

Compact set: - If $S \subseteq \mathbb{C}$ is | closed & bounded set, then it is called compact set.

Connect ted set: - A set $S \subseteq \mathbb{C}$ is said to be connected if any two points | z_1 and z_2 in S can be join by a(continuous|curve in S .

Domain: - non-empty + open + connected.

Simply connected domain: - domain with no holes

Complex Analysis

Multiconnected domain: - not simply connected

$\Rightarrow$ multiconnected domain

Region: - Domain with some none or all of its boundary points is said to be a region.

Q. What is an open disc (or open ball)?

(a) A set of points on the boundary of a circle.

(b) A set of points inside a circle excluding the boundary.

(c) A set of points outside a circle.

(d) A set of points that includes both the inside and the boundary of a circle.

Q. What defines a closed disc (or closed ball)?

(a) Only the boundary of a circle.

(b) Points outside the boundary of a circle.

(c) The interior points of a circle excluding the boundary.

(d) The interior points and the boundary of a circle.

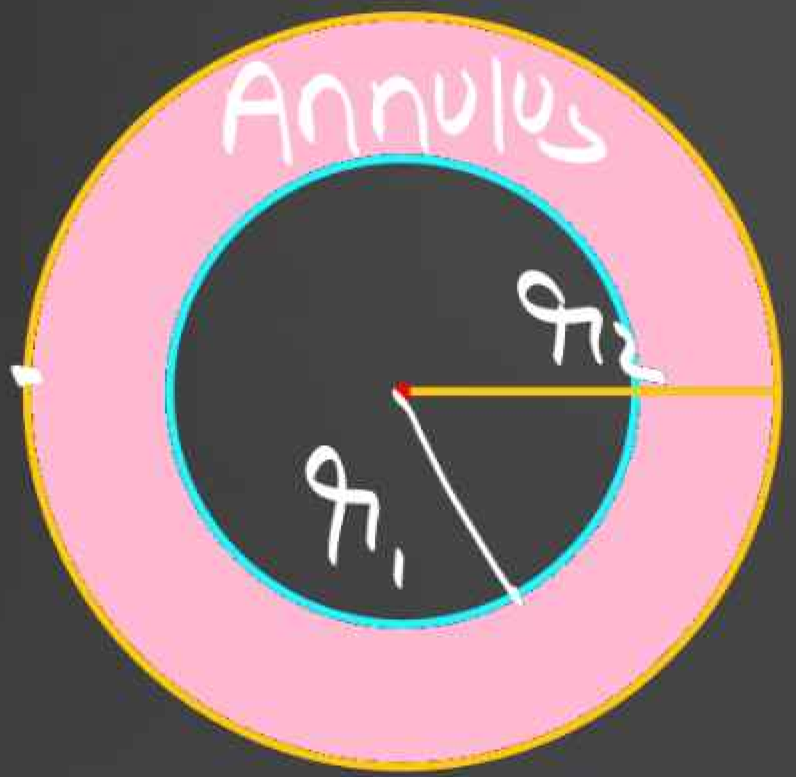
Q. Which of the following describes an annulus?

(a) A solid circle.

(b) The region between two concentric circles.

(c) A point on the boundary of a circle.

(d) A region outside a circle.



Q. What is a neighborhood of a point in complex analysis?

(a) A point itself.

(b) A set of points at a fixed distance from the point.

(c) A set containing the point and all points within some positive distance from it

(d) A set of points outside a given distance from the point.

Q. Which of the following is an interior point of a set S in the complex plane ?

(a) A point where every neighborhood contains points not in S.

(b) A point where some neighborhood is entirely contained in S.

(c) A point on the boundary of S.

(d) A point not in S.

Q. What is a boundary point of a set S ?

(a) A point where every neighborhood intersects both S and its complement.

(b) A point entirely within S.

(c) A point entirely outside S.

(d) A point not relevant to the set S .

Complex Function

Complex Function: - A function $f: D \subseteq \mathbb{C} \to \mathbb{C}$ is said to be a Complex Function

Complex Analysis

Trigonometric Function

Some Results: -

$$\sin^2 z + \cos^2 z = 1; \quad 1 + \tan^2 z = \sec^2 z$$

$$1 + \cot^2 z = \operatorname{cosec}^2 z; \quad \sin(-z) = -\sin z$$

$$\cos(-z) = \cos z; \quad \tan(-z) = -\tan z$$

Hyperbolic Function: -

$$\sinh\ z = \frac{e^z - e^{-z}}{2}; \cosh\ z = \frac{e^z + e^{-z}}{2}$$

$$\operatorname{sech}\ z = \frac{1}{\cosh\ z} = \frac{2}{e^z + e^{-z}}; \operatorname{cosech}\ z = \frac{1}{\sinh\ z} = \frac{2}{e^z - e^{-z}}$$

$$\tanh\ z = \frac{\sinh\ z}{\cosh\ z} = \frac{e^z - e^{-z}}{e^{-} + e^{-z}}; \coth\ z = \frac{\cosh\ z}{\sinh\ z} = \frac{e^z + e^{-z}}{e^z - e^{-z}}$$

$$\sinh\,(z_1 \pm z_2) = \sinh\ z_1 \cdot \cosh\ z_2 \pm \cosh\ z_1 \cdot \sinh\ z_2$$

$$\sin\,(iz) = i\sinh\ z; \cos\,(iz) = \cosh\ z$$

$$\tan\,(iz) = i\tanh\ z; \sinh\,(iz) = i\sin\ z$$

$$\cosh\,(iz) = \cos\ z; \tanh\,(iz) = i\tan\ z$$

$$\cosh^2\ z - \sinh^2\ z = 1$$

$$1 - \tanh^2\ z = \operatorname{sech}^2\ z$$

$$\coth^2\ z - 1 = \operatorname{cosech}^2\ z$$

Q. The value of $\cosh^2 z - \sinh^2 z$

$\cosh^2 z - \sinh^2 z$ **का मान-**

(a) $\cosh(2z)$

(b) 1

(c) $\sinh(2z)$

(d) 0

Q. The Real part of $\sin(x+iy)$ –

$\sin(x+iy)$ **का वास्तविक भाग है**

(a) $\sin x$

(b) $\cos y$

(c) $\sin x\cos hy$

(d) $\sin x\sin hy$

Q. What will be the imaginary/part of $\cosh(\alpha + i\beta)$ **?**

$\cosh(\alpha + i\beta)$ **का काल्पनिक भाग होगा?**

(a) $\cosh\ \alpha\cos\ \beta$

(b) $\sinh\ \alpha\sin\ \beta$

(c) $-\sin\ h\alpha\sin\ \beta$

(d) $-\cos\ h\alpha\cos\ \beta$

Logarithmic Function:

Q. $\log(1+i)$ **is equal to :/log** $(1+i)$ **बराबर है-**

(a) $\frac{1}{2}\log 2$

(b) $\frac{\pi}{4}+\frac{1}{2}\log 2$

(c) $\frac{1}{2}\log 2 + i\frac{\pi}{4}$

(d) None of these

$e^z = w$

$\log z = \log|w| + i(\arg w + 2n\pi)$

Q. Find out the all values of Z from this equation:

$e^z = 1 + \sqrt{3}i.$

a. $\ln(2) - (\pi \div 2 + 2\pi n)i; n \in Z$

b. $\ln(2) - (\pi \div 3 + \pi n)i; n \in Z$

c. $\ln(2) + (\pi \div 2 + \pi n)i; n \in Z$

d. $\ln(2) + (\pi \div 3 + 2\pi n)i; n \in Z$

Inverse Trigonometric Function:

$$\sin^{-1} z = \frac{1}{i}\ln\left(iz + \sqrt{1 - z^2}\right) \Rightarrow \frac{1}{i}\log\left(iz + \sqrt{1-z^2}\right)$$

$$\operatorname{cosec}^{-1} z = \frac{1}{i}\ln\left(\frac{i+\sqrt{z^2-1}}{z}\right)$$

$$\cos^{-1} z = \frac{1}{i}\ln\left(z + \sqrt{z^2 - 1}\right)$$

$$\sec^{-1} z = \frac{1}{i}\ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$

$$\tan^{-1} z = \frac{1}{2}\ln\left(\frac{1+iz}{1-iz}\right)$$

$$\cot^{-1} z = \frac{1}{2i}\ln\left(\frac{z+i}{z-i}\right) \Rightarrow \frac{1}{2i}\log\left(\frac{z+i}{z-i}\right)$$

Inverse Hyperbolic Function:

$$\sinh^{-1} z = \ln\left(z + \sqrt{z^2 + 1}\right) = \log\left(z + \sqrt{z^2+1}\right)$$

$$\operatorname{cosech}^{-1} z = \ln\left(\frac{1+\sqrt{z^2-1}}{z}\right)$$

$$\cosh^{-1} z = \ln\left(z + \sqrt{z^2 - 1}\right)$$

$$\operatorname{sech}^{-1} z = \ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$

$$\tanh^{-1} z = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$$

$$\coth^{-1} z = \frac{1}{2}\ln\left(\frac{z+1}{z-1}\right)$$

$$\operatorname{sech}^{-1} z = \log\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$

$$\operatorname{sech}^{-1}\left(\frac{1}{2}\right) = \log\left(\frac{1+\sqrt{1-\frac{1}{4}}}{\frac{1}{2}}\right)$$

$$\operatorname{sech}^{-1}\left(\frac{1}{2}\right) = \log\left(2+\sqrt{3}\right)$$

$$= \log\left(2\pm\sqrt{3}\right)$$

Q. $\operatorname{sech}^{-1}\left(\frac{1}{2}\right)$ **is equal to :**

$\operatorname{sech}^{-1}\left(\frac{1}{2}\right)$ **बराबर है:**

(a) $\log(\sqrt{3}\pm\sqrt{2})$

(b) $\log(\sqrt{3}\pm 1)$

(c) $\log(2\pm\sqrt{3})$ ✓

(d) इनमें से कोई नहीं

Principal Logarithm Function:

Q. The principal value of $\log\left(i^{1/4}\right)$ is

(a) πi

(b) $\frac{\pi}{2} i$

(c) $\frac{\pi}{4} i$

(d) $\frac{\pi}{8} i$

Principal Logarithmic fn →

$$\log z = \log|z| + i\arg(z)$$

$$\log i^{\frac{1}{4}}$$

$$\frac{1}{4}\log(0+i) = \frac{1}{4}\left[\log|0+i| + i\arg(0+i)\right]$$

$$\Rightarrow \frac{1}{4}\left[0 + i\tan^{-1}\left|\frac{1}{0}\right|\right] \Rightarrow \frac{1}{4}\left[i\cdot\frac{\pi}{2}\right] = \frac{\pi}{8} i$$

$$\frac{1}{0} = \infty$$

$$\tan\frac{\pi}{2} = \infty$$

(1) $\log(z_1 z_2) = \log z_1 + \log z_2$

(2) $\log\left(\frac{z_1}{z_2}\right) = \log z_1 - \log z_2$

(3) $\log\left(\frac{1}{z}\right) = \log 1 - \log z$

$\log\left(\frac{1}{z}\right) \Rightarrow -\log z$

Periods of Function: -

Q. i^i Find Principal Value?

(a) $e^{\frac{\pi}{2}}$

(b) $e^{-\frac{\pi}{2}}$

(c) $e^{-\frac{i\pi}{2}}$

(d) None of these.