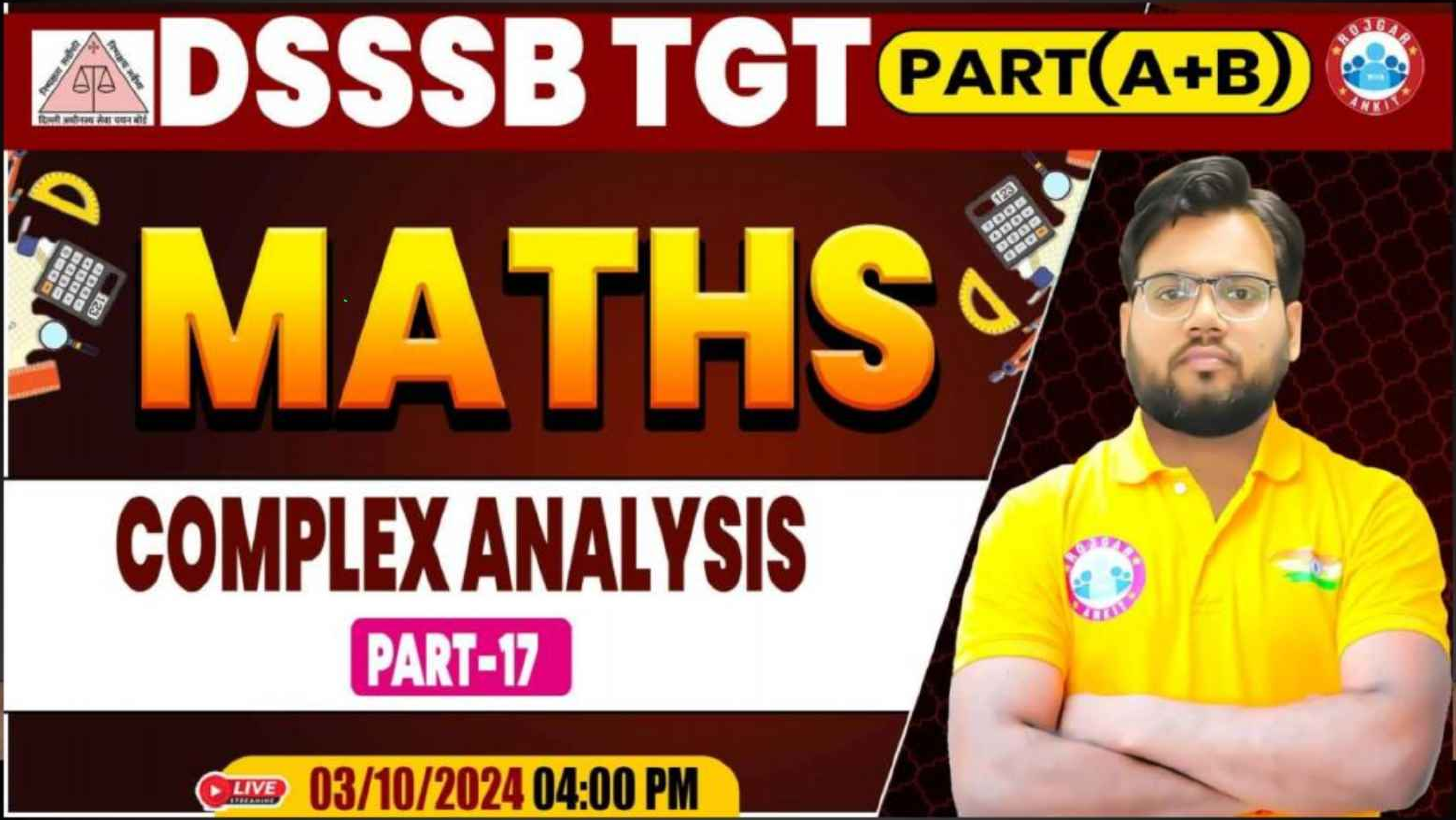
DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-17
LIVE
03/10/2024 04:00 PM

Limit of Complex Function: -

Set $f: D \subseteq \mathbb{C} \to \mathbb{C}$ **be a complex function and** $z_0 \in D$. **Then,** f **is said to have limit** l **if for given** $\varepsilon > 0$, $\exists \delta > 0$ **s.t.** $|f(z) - l| < \varepsilon, |z - z_0| < \delta, z \in$ **D.**

$$\lim_{z \to z_0} f(z) = u(x,y) + iv(x,y)$$

Sequential Criteria of limit:-

$f: D \subseteq \mathbb{C} \to \mathbb{C}$.

Then, the following are equivalent.

Algebra of limits:- Suppose $\lim_{z\to z_0} f(z) = A \& \lim_{z\to z_0} g(z) = B$

$\lim_{z\to z_\infty} f(z) = A$

$\lim_{z\to z_0} g(z) = B$

Then,

(1) $\lim_{z\to z_0}\{f(z) \pm g(z)\} = \lim_{z\to z_0} f(z) \pm \lim_{z\to z_0} g(z)$

$(A \pm B)$

(2) $\lim_{z\to z_0}\{f(z)\cdot g(z)\} =$

$\lim_{z\to z_0} f(z) \cdot \lim_{z\to z_0} g(z)$

$A \cdot B$

(3) $\lim_{z\to z_0} \frac{f(z)}{g(z)} =$

$$\frac{\lim_{z\to z_0} f(z)}{\lim_{z\to z_0} g(z)} = \frac{A}{B}$$

Infinite limit: -

(1) $\lim_{z\to\infty} f(z) = \infty$ **iff**

(2) $\lim_{z\to l} f(z) = \infty$ **iff**

(3) $\lim_{z\to\infty} f(z) = l$ **iff**

(4) $\lim_{z\to\infty} f(z) =$

Q. $\lim_{z\to 0} \frac{\text{Re}(z)}{\text{Im}(z)}$?

A) ∞

B) 0

C) 1

D) Does not exist

$z = 0$
$\Rightarrow x + iy = 0$
$\Downarrow \quad \Downarrow$
$0 \quad 0$

$z = \overset{\text{Re}}{x} + i\overset{\text{Im}}{y}$

Real $\Rightarrow x$
Imaginary $= y$

$$\lim_{z\to 0} \frac{\text{Re}(z)}{\text{Im}(z)}$$

$$\lim_{z\to 0} \frac{x}{y}$$

$$\boxed{y = mx}$$

$$\lim_{z\to 0} \frac{\cancel{x}}{m\cancel{x}}$$

$$\lim_{z\to 0} \frac{1}{m}$$

Limit depends on m

Q. $\lim_{z\to 0} z^2 + \bar{z}^2$

A) 0

B) 1

C) -1

D) Does not exist

$z = x + iy$

$\bar{z} = x - iy$

$$\lim_{z\to 0} z^2 + \bar{z}^2$$

$$\lim_{z\to 0}\left[(x+iy)^2 + (x-iy)^2\right]$$

$$= \lim_{z\to 0}\left[x^2 - y^2 + \cancel{2xiy} + x^2 - y^2 \cancel{- 2xiy}\right]$$

$$\Rightarrow \lim_{z\to 0}\left[2(x^2 - y^2)\right]$$

$$\Rightarrow \lim_{\substack{x\to 0\\ y\to 0}}\left(2(x^2-y^2)\right) \Rightarrow 0$$

$z = x + iy$

$0 = x + iy$

$\Downarrow \quad \Downarrow$

$0 \quad 0$

$z = 0$

$x = 0 \quad y = 0$

$$\lim_{z\to 0}\left(\frac{x^2}{x^2+y^2}+2i\right)$$

$\boxed{\text{put } y=mx}$

$$\lim_{z\to 0}\left(\frac{\cancel{x^2}}{\cancel{x^2}(1+m^2)}+2i\right)$$

$$\lim_{z\to 0}\left(\frac{1}{1+m^2}+2i\right)$$

$\Rightarrow$ Limit does not exit

Q. $\lim_{z\to 0}\left(\frac{x^2}{x^2+y^2}+2i\right)$

A) 0

B) $2i$

C) 1

D) Does not exist

$z = x+iy$

$z = 0$

तो $x=0, y=0$

Q. Evaluate $\lim_{z\to i}\left(\frac{z^2-1}{z^2+1}\right)$

A) 0

B) $\frac{-1}{2}$

C) 1

D. Does not exist

$$\lim_{z\to i}\left(\frac{z^2-1}{z^2+1}\right)$$

$$= \frac{-1-1}{-1+1} \Rightarrow \frac{-2}{0}$$

$$\lim_{z\to i}\left(\frac{\cancel{z^2}\left(1-\frac{1}{z^2}\right)}{\cancel{z^2}\left(1+\frac{1}{z^2}\right)}\right)$$

$$\Rightarrow \frac{1-\frac{1}{(-1)}}{1+\frac{1}{-1}} \Rightarrow \frac{1+1}{1-1} = \frac{2}{0}$$

Q. Evaluate $\lim_{z\to\infty}\left(\frac{z^2-2z+1}{z^2+2z+1}\right)$

A) 0

B) 1

C) ∞

D) -1

$$\lim_{z\to\infty}\left(\frac{z^2-2z+1}{z^2+2z+1}\right)$$

$$= \lim_{z\to\infty}\left(\frac{\cancel{z^2}\left(1-\frac{2}{z}+\frac{1}{z^2}\right)}{\cancel{z^2}\left(1+\frac{2}{z}+\frac{1}{z^2}\right)}\right)$$

$$\Rightarrow \frac{1-\frac{2}{\infty}+\frac{1}{\infty}}{1+\frac{2}{\infty}+\frac{1}{\infty}} \Rightarrow \frac{1-0+0}{1+0+0} = \frac{1}{1}$$

Q. Determine $\lim_{z\to 1}\left(\frac{z^3-1}{z-1}\right)$

A) 0

B) 1

C) 3

D) 2

$$\lim_{z\to 1}\left(\frac{z^3-1^3}{z-1}\right)$$

$$= \lim_{z\to 1}\left(\frac{\cancel{(z-1)}(z^2+1+z)}{\cancel{(z-1)}}\right)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$z^3-1^3=(z-1)(z^2+1+z)$$

$$\Rightarrow \lim_{z\to 1}\left(z^2+z+1\right)$$

$$\Rightarrow 1+1+1=3$$

Complex Continuous Function:- $f: D \subseteq \mathbb{C} \to \mathbb{C}$.

Then, $f(z) = u(x, y) + iv(x, y)$ **is continuous at** $z_0 = (x_0, y_0) \Leftrightarrow u(x, y) \& v(x, y)$ **are continuous at** $z_0 = (x_0, y_0)$

Algebra of Continuous function:- **Given** $f(z)$ **and** $g(z)$ **are continuous at** $z = z_0$**. Then, the function** $f(z) + g(z), f(z) - g(z), f(z) \cdot g(z)$ **and** $\frac{f(z)}{\gamma(z)}$ Continuously