DSSSB TGT PART(B)

MATHS

MULTIVARIATE CALCULUS

Part -5

LIVE 03/10/2024 08:00 AM

Question

Q. If $\vec{r} = (t+1)\hat{i} + (t^2+t+1)\hat{j} + t^3\hat{k}$, then $\frac{d\vec{r}}{dt}$ at $t = -1$, is equal to

यदि $\vec{r} = (t+1)\hat{i} + (t^2+t+1)\hat{j} + t^3\hat{k}$, तो t=-1 पर $\frac{d\vec{r}}{dt}$ किसके बराबर है

a) $\hat{i} + \hat{j} + 3\hat{k}$

b) $\hat{i} - \hat{j} + 3\hat{k}$

c) $\hat{i} - \hat{j} - 3\hat{k}$

d) $\hat{i} + \hat{j} - 3\hat{k}$

$\frac{d\vec{r}}{dt} \Rightarrow (1+0)\hat{i} + (2t+1)\hat{j} + 3t^2\hat{k}$

$t = -1$

$\frac{d\vec{r}}{dt} = \hat{i} - \hat{j} + 3\hat{k}$

Q. A particle moves along the curve $x = 3t^2$, $y = t^2 - 2t, z = t^3$, then its velocity at $t = 1$ is

एक कण वक्र $x = 3t^2, y = t^2 - 2t, z = t^3$ के अनुदिश गति करता है, तो t=1 पर इसका वेग क्या है?

a) $6\hat{\imath} + 3\hat{k}$

b) $3\hat{\imath} + 6\hat{k}$

c) $6\hat{\imath} - 3\hat{k}$

d) $2\hat{\imath} + 3\hat{\jmath}$

$v \Rightarrow x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{v} \Rightarrow 3t^2\hat{i} + (t^2 - 2t)\hat{j} + t^3\hat{k}$.

$\frac{d\vec{v}}{dt} = 6t\hat{i} + (2t - 2)\hat{j} + 3t^2\hat{k}$.

$\left(\frac{d\vec{v}}{dt}\right)_{t=1} \Rightarrow 6\hat{i} + 0\hat{j} + 3\hat{k}$

Q. A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$, then its component of acceleration at $t = 1$ in the direction of $\hat{i} + \hat{j} + 3\hat{k}$ is

एक कण वक्र $x = t^3 + 1, y = t^2, z = 2t + 5$ के अनुदिश गति करता है, तो t=1 पर $\hat{i} + \hat{j} + 3\hat{k}$ की दिशा में त्वरण का उसका घटक क्या है?

a) $\frac{6}{\sqrt{11}}$

b) $\frac{7}{\sqrt{11}}$

c) $\frac{8}{\sqrt{11}}$

d) $\frac{5}{\sqrt{11}}$

$\vec{v} \Rightarrow (t^3+1)\hat{i} + t^2\hat{j} + (2t+5)\hat{k}$

$\frac{d\vec{v}}{dt} = 3t^2\hat{i} + 2t\hat{j} + 2\hat{k}$

$\frac{d^2\vec{a}}{dt^2} \Rightarrow 6t + 2\hat{j} + 0\hat{k}$

$\Rightarrow \hat{i} + \hat{j} + 3\hat{k}$

$\frac{d^2\vec{a}}{dt^2} \cdot \vec{a}$

$\Rightarrow \frac{6t+2}{\sqrt{t^2+t^2+3^2}} \Rightarrow \frac{6+2}{\sqrt{11}}$

$= \frac{8}{\sqrt{11}}$

Q. If $\phi(x,y,z) = x^2y + y^2x + z^2$, then grad ϕ at the point $(1,1,1)$ is

यदि $\phi(x,y,z) = x^2y + y^2x + z^2$, तो बिंदु (1,1,1) पर $grad\ \phi$ है

a) $3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

b) $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

c) ~~$3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$~~

d) $\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$$\nabla\phi \Rightarrow \frac{d\phi}{dx}\cdot i + \frac{d\phi}{dy}\cdot j + \frac{d\phi}{dz} + \vec{k}$$

$$\frac{d\phi}{dx} \Rightarrow 2xy+y^2 \quad \frac{d\phi}{dy} \Rightarrow x^2+2xy \quad , \quad \frac{d\phi}{dz} \Rightarrow 0+0+2z$$

$$(\nabla\phi)_{(1,1,1)} \Rightarrow (2xy+y^2)\,i + (x^2+2xy)\,j + 2z\,\hat{k}$$

$$\Rightarrow 3i + 3j + 2k \;//$$

Q. The unit normal vector to the surface $x^4 - 3xyz + z^2 + 1 = 0$ at the point $(1,1,1)$ is

बिंदु (1,1,1) पर सतह $x^4 - 3xyz + z^2 + 1 = 0$ के लिए इकाई सामान्य वेक्टर है

a) $\frac{1}{\sqrt{11}}(\hat{i} + 3\hat{j} - \hat{k})$

b) $\frac{1}{\sqrt{13}}(\hat{i} - 3\hat{j} + \hat{k})$

c) $\frac{1}{\sqrt{13}}(\hat{i} - 3\hat{j} - \hat{k})$

d) $\frac{1}{\sqrt{11}}(\hat{i} - 3\hat{j} - \hat{k})$

$$\nabla\phi = \frac{d\phi}{dx}, \frac{d\phi}{dy}, \frac{d\phi}{dz}$$

$$\nabla\phi \Rightarrow (4x^3 - 3yz)\hat{i} - 3xz\hat{j} - 3xy + 2z\hat{k}$$

$$\nabla\phi \Rightarrow \hat{i} - 3\hat{j} - \hat{k}$$

$$\frac{\nabla\phi}{|\nabla\phi|} = \frac{\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{1+9+1}} \Rightarrow \frac{\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{11}}$$

Q. The directional derivative of $\phi(x,y,z) = x^2yz + 4xz^2$ at the point $(1,-2,1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ is

$\phi(x,y,z) = x^2yz + 4xz^2$ का दिशात्मक व्युत्पन्न बिंदु (1,-2,1) पर दिशा $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ में है

a) $\frac{13}{3}$

b) $-\frac{11}{3}$

c) $\frac{11}{3}$

~~d)~~ $-\frac{13}{3}$

$\nabla\phi \cdot \hat{a}$ → Unit vector.

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \Rightarrow \frac{2i - j - 2k}{\sqrt{4+1+4}} \Rightarrow \frac{2i - j - 2k}{3}$

$\nabla\phi = (2xyz + 4z^2)\hat{\imath} + x^2z\,\hat{\jmath} + (x^2y + 8xz)\hat{k}$

$\nabla\phi = \hat{\jmath} + 6\hat{k}$

$\nabla\phi = \hat{\jmath} + 6\hat{k}$

$\nabla\phi \cdot \vec{a}$ ka $= \frac{-1-12}{3} = \frac{-13}{3}$

Q. The equation of tangent Plane to the surface $x^2 + y^2 + z^2 = 25$ at the point $(4,0,3)$ is

बिंदु $(\hat{4},\hat{0},\hat{3})$ पर सतह $x^2 + y^2 + z^2 = 25$ के स्पर्शरिखा तल का समीकरण है

a) $4X + 3Z - 25 = 0$

b) $3X + 4Z - 25 = 0$

c) $4X + 3Y - 25 = 0$

d) $4Y + 3Z - 25 = 0$

$$(x-x)\frac{d\phi}{dx} + (y-y)\frac{d\phi}{dy} + (z-z)\frac{d\phi}{dz} = 0$$

$$(x-4)\,8 + (y-0).0 + (z-3).6 = 0$$

$$8x - 32 + 6z - 18 = 0$$

$$8x + 6z - 50 = 0$$

$$\boxed{4x + 3z - 25 = 0}$$

$$\frac{d\phi}{dx} \Rightarrow 2x \to 8$$

$$\frac{d\phi}{dy} = 2y \to 0$$

$$\frac{d\phi}{dz} = 2z \to 6$$

Q. The equation of normal to the surface $z = x^2 + y^2$ at the point $(2, -1, 5)$ is

बिंदु $(2, -1, 5)$ पर सतह $z = x^2 + y^2$ के अभिलम्ब का समीकरण है

a) $\frac{X-2}{4} = \frac{Y+1}{-2} = \frac{Z-5}{-1}$

b) $\frac{X-2}{4} = \frac{Y+1}{2} = \frac{Z-5}{1}$

c) $\frac{X+2}{-4} = \frac{Y-1}{2} = \frac{Z+5}{1}$

d) None of these

$x^2+y^2-z = 0$

$\frac{d\phi}{dx} = 2x$, $\frac{d\phi}{dy} = 2y$, $\frac{d\phi}{dz} = -1$

$\frac{x-x_1}{\frac{d\phi}{dx}} = \frac{y-y_1}{\frac{d\phi}{dy}} = \frac{z-z_1}{\frac{d\phi}{dz}}$

$\frac{x-2}{4} = \frac{y+1}{-2} = \frac{z-5}{-1}$

$y^2 + z^2 = 29$ **and** $x^2 + y^2 + z^2 + 4x - 6y - 8z - 4 = 0$ **is**

ीं $x^2 + y^2 + z^2 = 29$ और $x^2 + y^2 + z^2 + 4x - 6y - 8z - 4 = 0$ के

बिंदु (4,-3,2) पर प्रतिच्छेद कोण क्या है?

$\cos^{-1}\sqrt{\frac{17}{29}}$

$\cos^{-1}\sqrt{\frac{19}{29}}$

$\cos^{-1}\sqrt{\frac{23}{29}}$

$\cos^{-1}\sqrt{\frac{14}{29}}$

$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1| \cdot |\nabla\phi_2|}$

$\nabla\phi_1 \Rightarrow 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

$\nabla\phi_1 = 8\hat{i} - 6\hat{j} + 4\hat{k}$

$|\nabla\phi_1| = \sqrt{64+36+16} = \sqrt{116}$

$\nabla\phi_2 = (2x+4)\hat{i} + (2y-6)\hat{j} + (2z-8)\hat{k}$

$\nabla\phi_2 = 12\hat{i} - 12\hat{j} - 4\hat{k}$

$|\nabla\phi_2| = \sqrt{144+144+16} = \sqrt{304}$

$\cos\theta = \frac{96+72-16}{\sqrt{116}\cdot\sqrt{304}}$

$\cos\theta \Rightarrow \frac{66+96}{\sqrt{116\times 304}}$

$\cos\theta = \frac{152}{2\sqrt{29}\times 4\sqrt{19}}$ $\quad 38 \quad \sqrt{19} \quad 19$

Q. The greatest rate of increase of $u = xyz^2$ at the point $(1,0,3)$ is

बिंदु (1,0,3) पर $u = xyz^2$ की वृद्धि की सबसे बड़ी दर क्या है?

a) 6

b) 9

c) 7

d) 5

Home-work