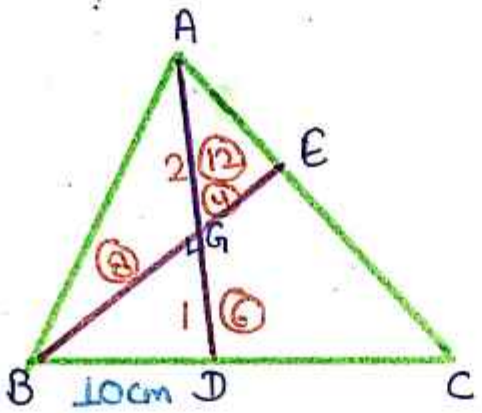
GEOMETRY

CENTRES OF TRIANGLE

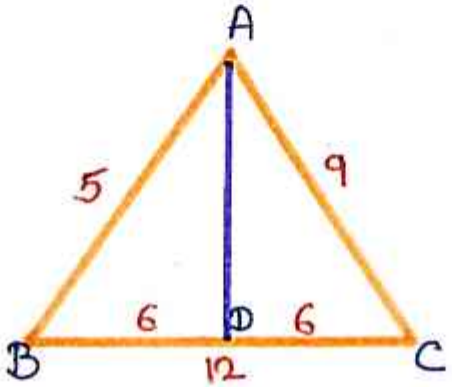
Q) In ΔABC, two medians AD and BE intersect at right angles at G. If AD = 18cm and BE = 12 cm, then the length of BD is equal to?

ΔABC में, दो माध्किाएँ AD और BE, G पर समकोण पर प्रतिच्छेद करती हैं। यदि AD = 18 cm और BE = 12 cm है। तो BD की लंबाई किसके बराबर है?



Q) In ΔABC, AB, BC and AC are equal to 5 cm, 12 cm and 9cm respectively. If the median AD intersects the opposite side BC at D, find the measure of the median AD.

ΔABC में, AB, BC और AC क्रमशः 5 सेमी, 12 सेमी और 9 सेमी के बराबर हैं। यदि माध्यिका AD. विपरीत भुजा BC को D पर प्रतिच्छेदित करती है, तो माध्यिका AD. की माप ज्ात कीजिए।



$5^2+9^2=(6^2+AD^2)2$

$25+81$

$106 = (36+AD^2)\times 2$

53

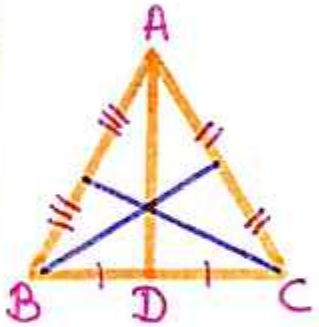
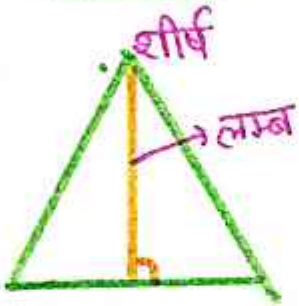
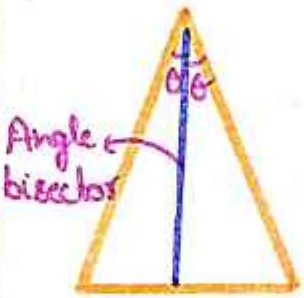
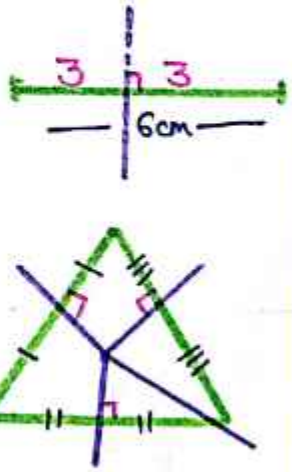
$AD^2 = 17$

$AD = \sqrt{17}$ cm

INCENTER / EX-CENTER / CIRCUMCENTER / ORTHOCENTER

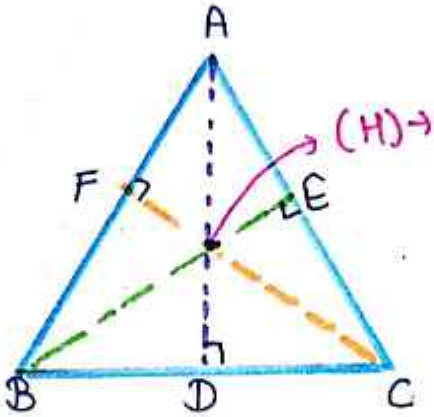
-:- CENTER (केन्द्र) -:-

1. Centroid (केन्द्रक)
2. Orthocenter (लम्बकेन्द्र)
3. Circumcenter (परिकेन्द्र)
4. Incenter (अंत: केन्द्र)
5. Excenter (बाह्यकेन्द्र)

MEDIAN माध्यिका	ALTITUDE लम्ब	ANGLE BISECTOR कोण द्विभाजक	PERPENDICULAR BISECTOR लम्ब द्विभाजक
			

ORTHOCENTER (लम्ब केन्द्र)

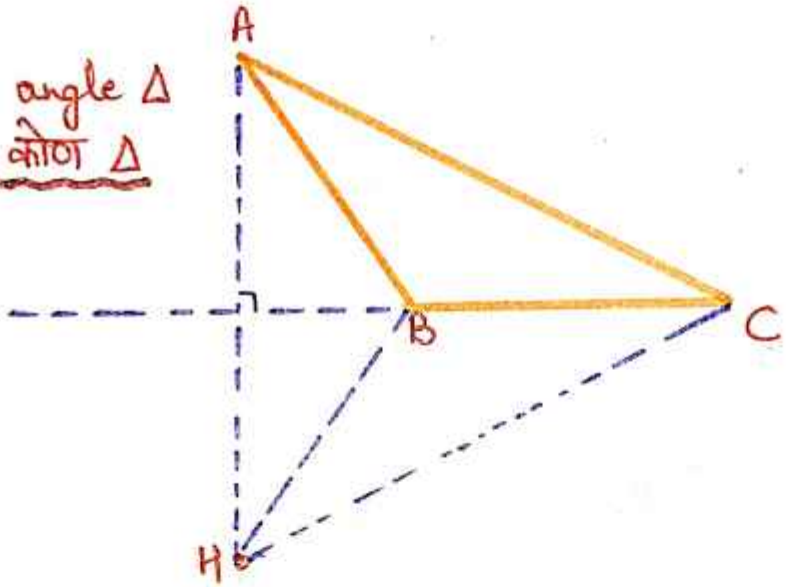
* Acute Angle Δ
न्यून कोण Δ



$$\begin{bmatrix} AD \perp BC \\ BE \perp AC \\ CF \perp AB \end{bmatrix}$$

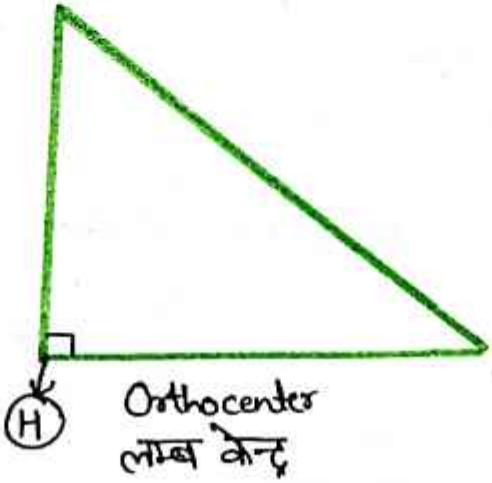
• Where all three altitudes meet → Orthocenter
जहाँ तीनों लम्ब मिलते है → लम्ब केन्द्र

* Obtuse angle Δ
अधिक कोण Δ



• Orthocenter of obtuse angle Δ lies outside the triangle
अधिक कोण त्रिभुज का लम्ब केन्द्र त्रिभुज के बाहर होता है।

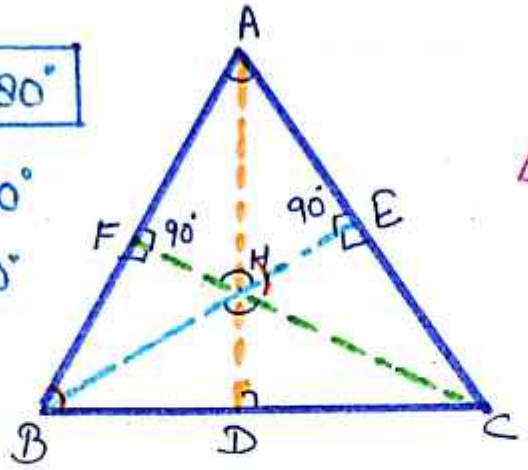
* Right angle Δ
समकोण त्रिभुज



$\boxed{\angle BHC + \angle A = 180^\circ}$

$\angle CHA + \angle B = 180^\circ$

$\angle AHB + \angle C = 180^\circ$

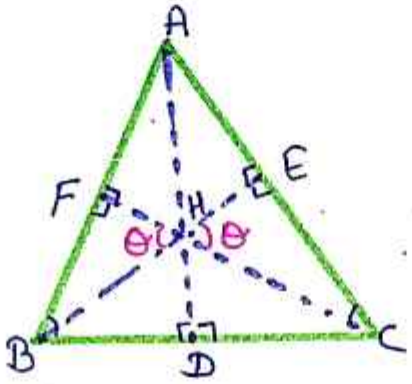


$\angle BHC = \angle FHE$
opposite angles

$\square FAEH$

$90 + A + 90 + \angle EHF = 360$

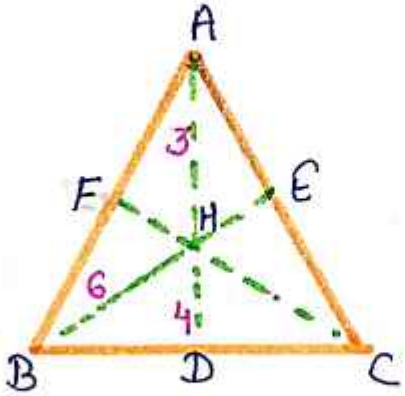
$\boxed{A + \angle EHF = 180^\circ}$



$\Delta FHB \sim \Delta EHC$

$\frac{HB}{HC} = \frac{FH}{EH} \Rightarrow \boxed{HB \times EH = FH \times HC = AH \times HD}$

Ex:-



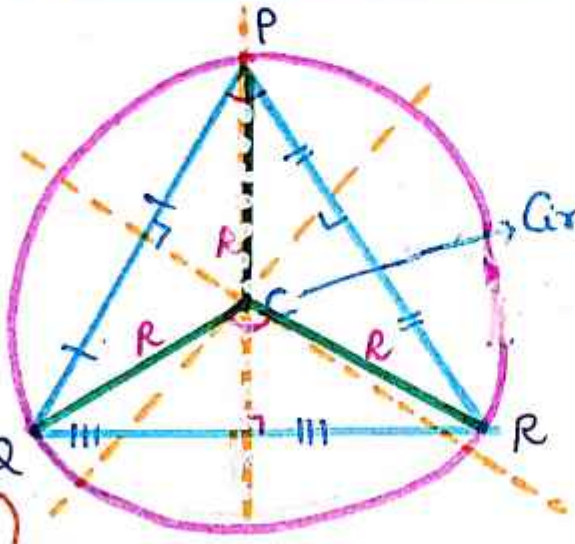
HE = ?

$AH \times HD = BH \times HE = CH \times HF$

$3 \times 4 = 6 \times HE$

$HE = \frac{12}{6}$ ②

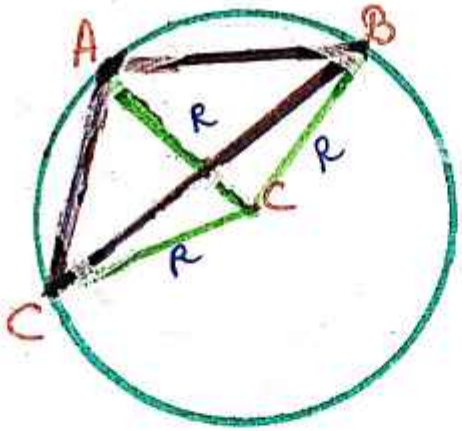
CIRCUMCENTER (परिकेन्द्र)

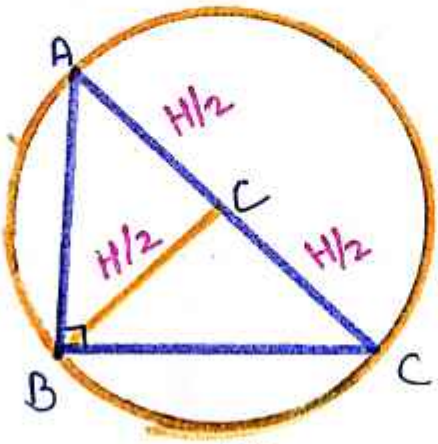


Circumcircle (परिकेन्द्र)

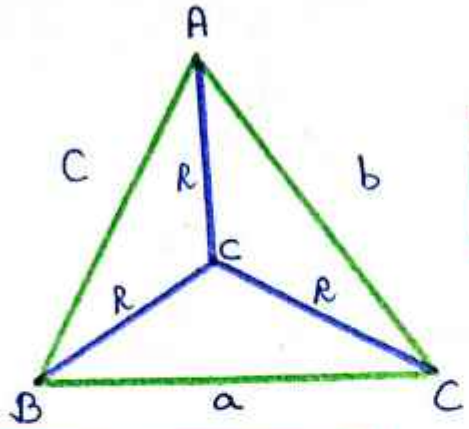
R = Circumradius (परित्रिज्या)

$$\begin{cases} \angle QCR = 2\angle P \\ \angle RCP = 2\angle Q \\ \angle QCP = 2\angle R \end{cases}$$





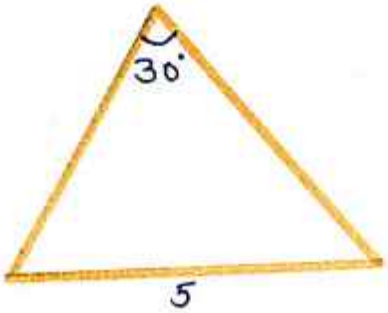
समकोण त्रिभुज का परिकेन्द्र = $\frac{\text{कर्ण}}{2}$



$$R = \frac{abc}{4\Delta}$$ → Δ → area of triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

R = परित्रिज्या



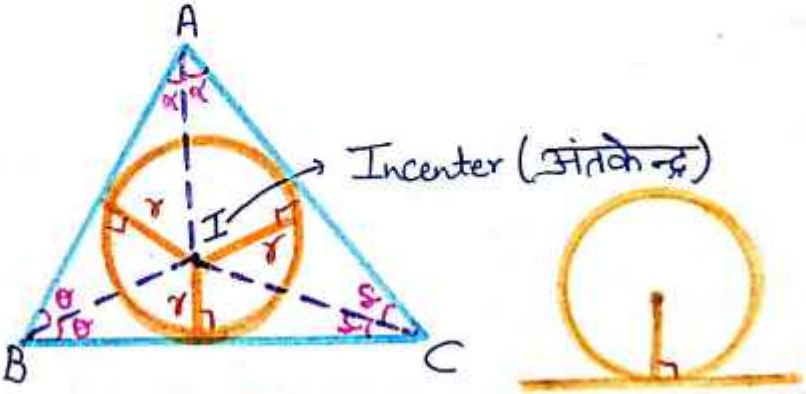
R = ?

$$\frac{a}{\sin A} = 2R$$

$$\frac{5}{\sin 30} = 2R$$

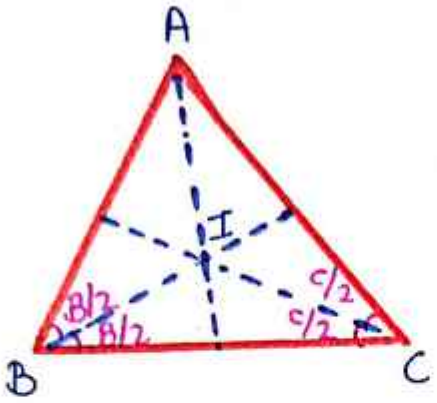
$$\frac{5}{1/2} = \frac{10}{5} = 2R \qquad R = 5$$

INCENTER (अंतकेन्द्र)



α → alpha
θ → Theta
γ → Gamma

Where all three angle bisectors meet → Incenter
जहाँ पर तीनो कोण द्विभाजक मिलते है → अंतकेन्द्र
r → inradius (अंत: त्रिज्या)



$\Delta BIC \Rightarrow \frac{B}{2}+\frac{C}{2}+\angle BIC = 180°$

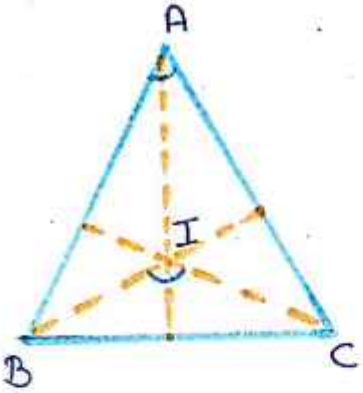
$\angle BIC = 180-\left(\frac{B+C}{2}\right)$

$\angle BIC = 180-\frac{1}{2}(180-A)$

$180-90+\frac{A}{2}$

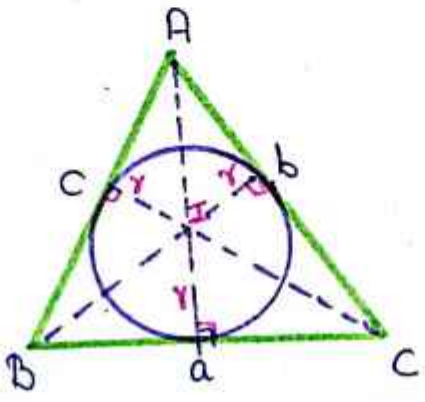
$\therefore A+B+C=180$
$B+C=180-A$

$$\boxed{\angle BIC = 90°+\frac{A}{2}}$$



$$\begin{cases} \angle BIC = 90+\frac{A}{2} \\ \angle CIA = 90+\frac{\angle B}{2} \\ \angle AIB = 90+\frac{C}{2} \end{cases}$$

$$\boxed{\Delta = r \times s}$$



s = अर्धपरिमाप

$s = \frac{a+b+c}{2}$

$$ar\,\Delta ABC = ar\,\Delta BIC + ar\,\Delta CIA + ar\,\Delta AIB$$

$$\downarrow \qquad \frac{1}{2} \times a \times r + \frac{1}{2} \times b \times r + \frac{1}{2} \times c \times r$$

$$\Delta = \frac{1}{2} r\,(a+b+c) \Rightarrow r \times \left(\frac{a+b+c}{2}\right) = r \times s$$