DSSSB TGT PART(A+B)

MATHS

COMPLEX ANALYSIS

PART-18

LIVE STREAMING 07/10/2024 04:00 PM

Limit of Complex Function: -

$f: D \subseteq \mathbb{C} \to \mathbb{C}$ **be a complex function and** $z_0 \in D$**.**
Then, f **is said to have limit** l **if for given** $\varepsilon > 0$**,**
$\exists \delta > 0$ **s.t.** $|f(z) - l| < \varepsilon, |z - z_0| < \delta, z \in$ **D.**

Sequential Criteria of limit:- $f: D \subseteq \mathbb{C} \to \mathbb{C}$.

Then, the following are equivalent.

Algebra of limits:- Suppose $\lim_{z \to z_0} f(z) = A \& \lim_{z \to z_0} g(z) = B$

Then,

(1) $\lim_{z \to z_0} \{f(z) \pm g(z)\} =$

(2) $\lim_{z \to z_0} \{f(z) \cdot g(z)\} =$

(3) $\lim_{z \to z_0} \frac{f(z)}{g(z)} =$

Infinite limit: -

(1) $\lim_{z\to\infty} f(z) = \infty$ **iff**

(2) $\lim_{z\to l} f(z) = \infty$ **iff**

(3) $\lim_{z\to\infty} f(z) = l$ **iff**

(4) $\lim_{z\to\infty} f(z) =$

Q. $\lim_{z \to 0} \frac{\mathrm{Re}\,(z)}{\mathrm{Im}\,(z)}$ **?**

A) ∞

B) 0

C) 1

D) Does not exist

Q. $\lim_{z \to 0} z^2 + \bar{z}^2$

A) 0

B) 1

C) -1

D) Does not exist

Q. $\lim_{z \to 0}\left(\frac{x^2}{x^2+y^2}+2i\right)$

A) 0

B) $2i$

C) 1

D) Does not exist

Q. Evaluate $\lim_{z\to i}\left(\frac{z^2-1}{z^2+1}\right)$

A) 0

B) $\frac{-1}{2}$

C) 1

D. Does not exist

Q. Evaluate $\lim_{z\to\infty}\left(\frac{z^2-2z+1}{z^2+2z+1}\right)$

A) 0

B) 1

C) ∞

D) -1

Q. Determine $\lim_{z \to 1} \left(\frac{z^3 - 1}{z - 1} \right)$

A) 0

B) 1

C) 3

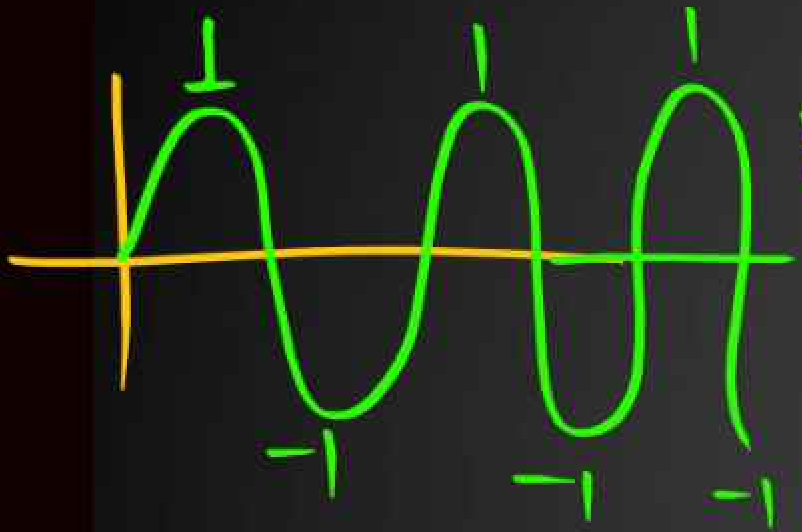
D) 2

Complex Continuous Function:- $f: D \subseteq \mathbb{C} \to \mathbb{C}$. **Then,** $f(z) = u(x,y) + iv(x,y)$ **is continuous at** $z_0 = (x_0, y_0) \Leftrightarrow u(x,y) \& v(x,y)$ **are continuous at** $z_0 = (x_0, y_0)$

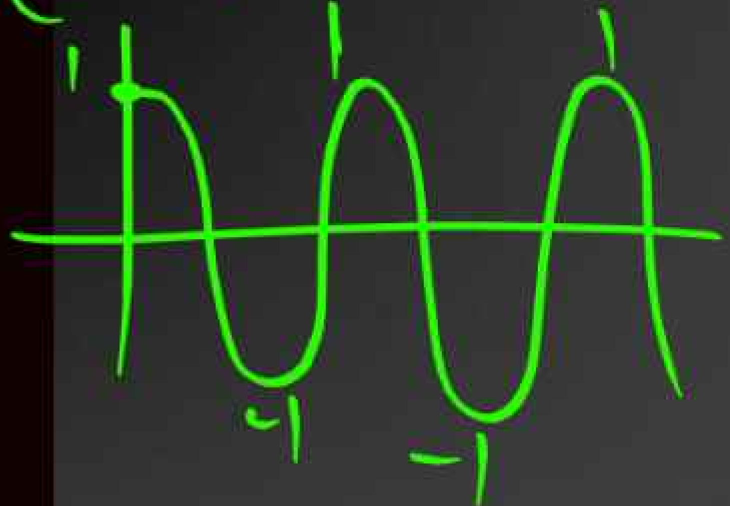
if & only if

Algebra of Continuous function:- **Given** $f(z)$ **and** $g(z)$ **are continuous at** $z = z_0$**. Then, the function** $f(z) + g(z), f(z) - g(z), f(z) \cdot g(z)$ **and** $\frac{f(z)}{g(z)}$ will be Continuous

Sin(x)



Cos x



Q. Determine whether the function $f(z) = \sin(z)$ is continuous in the complex plane.

A) $f(z) = \sin(z)$ **is continuous everywhere in the complex plane.**

B) $f(z) = \sin(z)$ **is continuous only on the real axis.**

C) $f(z) = \sin(z)$ **is continuous only on the imaginary axis.**

D) $f(z) = \sin(z)$ **is not continuous anywhere in the complex plane.**

$\sin(z) = \sin(x+iy)$

$\Rightarrow \sin x \cdot \cosh y + i \cdot \cos x \cdot \sinh y$

$\cos iy = \cosh y$

$\sin iy = i \sinh y$

$\cos(x+iy)$

$\cos x.\cosh y - i\sin x \sinh y$

Q. Determine whether the function' $f(z) = \cos(z)$ is continuous in the complex plane.

A) $f(z) = \sin(z)$ **is continuous everywhere in the complex plane.**

B) $f(z) = \sin(z)$ **is continuous only on the real axis.**

C) $f(z) = \sin(z)$ **is continuous only on the imaginary axis.**

D) $f(z) = \sin(z)$ **is not continuous anywhere in the complex plane.**

$f(z) = \frac{z^2+2z+1}{z^2-1}$

$\Rightarrow \frac{z^2+2z+1}{(z+1)(z-1)}$

$z \neq -1$

$z \neq 1$

Q. Determine whether the function $f(z) = \frac{p(z)}{q(z)}$ is continuous in the complex plane, where $p(z)$ and $q(z)$ are polynomials in $\mathbb{C}[x]$. consider the function: $f(z) = \frac{z^2+2z+1}{z^2-1}$.

(A) $f(z)$ is continuous everywhere in the complex plane.

B) $f(z)$ is continuous only where $q(z) \neq 0$.

C) $f(z)$ is continuous only on the real axis.

D) $f(z)$ is not continuous anywhere in the complex plane.

$f(z) = z$

$\Rightarrow x + iy$

Polynomial fun

Polynomial fun

Q. Determine whether the function $f(z) = z$ is continuous in the complex plane.

A) $f(z) = z$ is continuous everywhere in the complex plane.

B) $f(z) = z$ is continuous only on the real axis.

C) $f(z) = z$ is continuous only on the imaginary axis.

D) $f(z) = z$ is not continuous anywhere in the complex plane.

$a_0 + \underbrace{a_1 x^1}_{} + a_2 x^2$

1.Polynomials: - **Any polynomial function $p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0$, where a_i are complex coefficients, is continuous everywhere in the complex plane.**

x y

$a_0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots$

2. Rational Functions: - **A rational function $f(z) = \frac{p(z)}{q(z)}$, where $p(z)$ and $q(z)$ are polynomials and $q(z) \neq 0$, is continuous wherever $q(z) \neq 0$.**

$\frac{p(z)}{q(z)}$, $\boxed{q(z) \neq 0}$

3.Exponential Functions: - The exponential function e^z is continuous everywhere in the complex plane.

e^z,

Sec z
Cosec z
tan z
Cot z

4. Trigonometric Functions: - The trigonometric functions $\sin(z)$ and $\cos(z)$ are continuous everywhere in the complex plane.

tan hz
cot hz
sec hz
cosec hz

5. Hyperbolic Functions: - The hyperbolic functions $\sinh(z)$ and $\cosh(z)$ are continuous everywhere in the complex plane.

6.Logarithmic Functions: - The logarithmic function $\log(z)$ is continuous on its domain, which is the complex plane excluding the origin and any branch cuts.

7.Power Functions: - Functions of the form z^a, where a is a constant, are continuous where they are defined. For fractional powers, branch cuts are necessary.

$$\left(\frac{Z^2+Z}{Z^2}\right)$$

Z^2, $(x+iy)^2$

8.Roots: - Functions like $\sqrt{z}$ (square root) are continuous where they are defined, typically involving branch cuts to handle multivaluedness.

$\sqrt{-5}$ $\sqrt{-16} = 4i$ $\sqrt{-16} = \sqrt{-1} \times \sqrt{16}$

$i \times 4 \rightarrow 4i$

9. Complex Conjugation: - **The function $\bar{z}$, which gives the complex conjugate of z, is continuous everywhere in the complex plane.**

$z = a + ib$

$\bar{z} = a - ib$

10. Composition of Continuous Functions: - **If $f(z)$ and $g(z)$ are continuous functions, then $f(g(z))$ is also continuous.**

$f(z)$ - Cts

$g(z)$ - Cts

$f(g(z))$

$f(z) \times g(z)$ Cts

11. Absolute Value Functions: - The function $|z|$, which gives the magnitude of z, is continuous everywhere in the complex plane.

$|z|$

$\Rightarrow$